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The Mathematics Teacher

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Modern mathematics and the secondary school curriculum
E. P. NORTHROP

Where the mathematics teacher is king
PAUL R. NEUREITER

Where do eligible mathematics teachers go?
RAY C. MAUL

Practical mathematics is challenging to students
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Modern mathematics and the secondary school curriculum[1]

E. P. NORTHROP, *College of the University of Chicago, Chicago, Illinois.*

This article underlines the current dissatisfaction with the high school curriculum and, at the same time, offers a few pointers for the next steps to be taken.

INTRODUCTION

IN ACCORDING ME the honor of addressing you today, our president asked me to talk about modern mathematics and its effect on the secondary school curriculum. In accepting the invitation, I asked leave to omit the words "its effect on" from the title.

My request was prompted by a charge I had only recently encountered in a survey of secondary school mathematics prepared for the National Association of Secondary School Principals under the guidance of a committee of the National Council of Teachers of Mathematics.[2] In this volume there is a chapter on curriculum problems, and in this chapter a paper by Saunders MacLane on the impact of modern mathematics.[3] This paper begins with the proposition: "My subject is vacuous."

Now MacLane was in a position to go on and say why, as he does for several pages, because his audience was not present. But my audience is present, and my right to spend your time on a vacuous subject would be too obviously open to challenge from the floor. Having no wish to retire in confusion from your indignation at the beginning of what I have to say, I chose instead the title "Mathematics and the Secondary School Curriculum," feeling that the simple conjunction "and" would protect me from such a plight and give me the latitude I need.

I am anxious to avoid your indignation at the beginning of what I have to say because I am worried enough about how indignant you may be, for other reasons, at the end. For, taking my text from MacLane, I suggest that we teachers face up to the fact that secondary school mathematics has become, and is continuing to become, less and less relevant to modern mathematics and its application to modern science—physical, biological, and social. Specifically, I should like to examine some of the evidence, inquire into some of the causes, call attention to the need for change, and make some suggestions for bringing changes about.

SOME EVIDENCE

How can we best judge the relevance of secondary school mathematics to modern mathematics? Presumably by taking a look at both. This is more easily done in the case of the former than in the case of the latter, and for obvious reasons. There have been few attempts to describe modern mathematics in terms appropriate to a brief inquiry such as ours here today. A

[1] Presented at the Boston meeting of The National Council of Teachers of Mathematics, April 15, 1955.

[2] "Mathematics in Secondary Schools Today," *Bulletin of the National Association of Secondary School Principals*, Vol. 38, No. 203, May, 1954, pp. 1–196.

[3] S. MacLane, *ibid.*, pp. 66–70.

recent and laudable attempt has been made by the Committee on Training and Research in Applied Mathematics of the National Research Council, in an as yet unpublished report. With your permission I shall quote from this report at some length.

> The early 1900's saw the consummation of a subtle emancipation of mathematical thought which had been in the making for the better part of the previous century and which has profoundly changed the character of modern mathematics. . . .
>
> Traditionally, standard systems of mathematical axioms, such as those of Euclidean geometry or classical analysis, had been conceived as laying down basic self-evident truths. Their self-evidence was deemed well worth the effort of profound philosophical speculations, but the essential "rightness" of their consequences gave satisfactory proof of the world's ultimate orderliness about whose nature the human mind was evidently not deceived. Moreover, the incisive sweep of the resulting mathematical theorems, as well as the reliability, for example, with which its conclusions in the framework of Newtonian mechanics were corroborated by nature, appeared as cut from the same cloth. For the last fifty years, however, one has recognized systems of mathematical axioms as nothing more than the imaginative creations of the human mind, working in the medium of concept and symbol; instead of external a priori truths, the best that can be expected of them is consistency—and the a posteriori satisfaction if they turn out to do their job well.
>
> This had the effect of a major break-through, and since then mathematics has fanned out in ever new directions as new structures were postulated, examined, modified, related to others, and finally left standing in one or another definitive form. Sloughing off the classical cloak of an imbedding Cartesian continuum, algebra emerged in the abstract: groups, rings, vector spaces, fields, and algebras became axiomatically established entities. The sloughed-off continuum takes on an independent life as a topological space of elements without algebraic relations. Its axiomatizations launch another mighty branch of modern mathematics which, in its turn, splits once more into an algebraic combinatorial and a set-theoretical component. Even more general structures emerge on the side of abstract algebra in the theory of lattices and on the other in set theory—both of them beginning to show the bare bones of mathematical foundations. The latter, in their turn, become themselves the subject of intensive research, making use of the newly gained freedom and producing results with deeply perturbing implications for the entire venture of mathematics. Analysis, too, breaks its bonds as measure theory and integration are put on a general axiomatic foundation. The conception of Hilbert space imbeds classical Fourier analysis in a new, predominantly algebraic context and carries on to the study of even more general normed linear spaces. In addition, a theory of linear operators and their spectral resolution emerges in progressively more abstract terms. As a counter current to these developments of axiomatic analysis, where new structures are defined mainly by retaining only particularly selected features of the traditional mathematical objects, an equally important trend of axiomatic synthesis has gained momentum. It has brought forth creative conceptions of compelling power, illustrated, for instance, by the fusion of algebraic and analytical aspects in the general calculus of tensors, or—in other directions—the theory of topological groups and of similarly topologized algebraic structures, and finally, the ideas and results concerning the way in which the topological features of a manifold interact not only with its own local analytic structure but also with the properties of the differentials and integrals existing on it—ideas and results developed in the theory of harmonic integrals and its ramifications or, more generally, under the heading of analysis in the large.[4]

It is not necessary to understand all the technical terms used in the foregoing description to grasp its chief import, namely, that recent years have seen an explosive development of mathematical knowledge, and that in the course of the explosion the concepts and methods that in our days as students were neatly compartmentalized into algebra, geometry, and analysis have now become mixed beyond all recognition.

How have these extraordinary changes in the nature and structure of mathematics been reflected in the secondary curriculum? It is hard to find even a trace of a reflection.

Take, for example, two of the United States Office of Education's most recent publications about mathematics,[5] *Mathematics in the Public High Schools* and *Cur-*

[4] Committee on Training and Research in Applied Mathematics of the National Research Council, *Final Report on a Survey of Training and Research in Applied Mathematics in the United States* (mimeographed), November, 1954, pp. 8, 9. This report has been approved by the Council and transmitted to the National Science Foundation, under a grant from which the survey was conducted.

[5] *Mathematics in The Public High Schools*, U. S. Office of Education, Bulletin 1953, No. 5, Government Printing Office. *Curricular Materials in High School Mathematics*, U. S. Office of Education, Bulletin 1954, No. 9, Government Printing Office.

riculum Materials in High School Mathematics. The first of these publications deals with statistics on enrollments, class size and length, and requirements for graduation, all by subject and by grade level. The second deals with statistical comparisons of state and local curriculum guides of 1932 and 1952 in such terms as type and size of publication and with treatment of objectives, teaching procedures, and testing. However important such knowledge may be to some people and for some purposes, it is astonishing that the only references to *mathematics* are in terms of the conventional course labels, "algebra," "plane geometry," and the like. Perusal of these documents leaves me with the impression that the only substantial curricular change since I attended high school thirty years ago has been the introduction of certain courses called "general mathematics" and "consumer mathematics." It seems clear that neither of them was introduced as a result of changes at the frontiers of mathematics.

By and large, other publications such as those of The National Council of Teachers of Mathematics are not much more rewarding. Examination of such publications prior to 1950 reveals the incorporation into secondary mathematics of but one substantial modern mathematical concept since my high school days. I refer to the concept of function which, after almost as much abuse as use, is already out of date in the form in which it has finally won a place for itself in the curriculum.

Only in publications since 1950 are tenuous but encouraging hints of conceivable changes to be found. In the survey referred to earlier, for example—the survey conducted for the National Association of Secondary School Principals—there are stirrings other than the one raised by MacLane. However, the fact remains that with perhaps one discernable exception, the most radical proposals involve little more than the pruning of deadwood like solid geometry to make way for live but nevertheless conventional growths like analytic geometry, or calculus, or statistics. One is tempted to conclude, with MacLane, that "the lively modern development of mathematics has had no impact on the content or on the presentation of secondary school mathematics."[6]

Some causes

According to MacLane, the reason that secondary school mathematics has become isolated from modern mathematics is that, "The mathematicians have pursued the new ideas with single-minded devotion, while the teachers have concentrated on the reformulation and presentation of the old ideas, so that there has been no exchange of interests."[7]

One of the most stubborn obstacles to exchanges of interests is the snail's pace at which advances at the frontiers of mathematics are passed down the educational line to the public at large. This phenomenon for science in general has been noted by Alan Waterman, Director of the National Science Foundation, as follows:

> I have . . . remarked, as a rough generalization, that a highly significant discovery is confirmed, attested, and accepted by leaders in its special field perhaps within a decade; it becomes understood and accepted by the general body of scientists in that field within the generation; and some time during that century it will be generally accepted by literate mankind.[8]

In some ways this estimate is on the low side for mathematics: the value 3 was ascribed to π not only by the Old Testament, but by the state legislator who is said to have sought in recent years to introduce a bill designed to simplify computations for the young people of his state.

It is this time lag that is largely responsible for the failure of communication between mathematicians and teachers. Instruction in mathematics seems to be carried on in two nonintersecting vicious circles. In the upper and less penetrable

[6] MacLane, *loc. cit.*, p. 66.
[7] MacLane, *loc. cit.*, p. 67.
[8] A. T. Waterman, "Acceptance of Science," *The Scientific Monthly*, Vol. 80, No. 1 (January 1955), p. 10.

circle, new developments at the frontiers of mathematics are incorporated into advanced graduate courses in universities, where they are passed along to budding research mathematicians, some of whom become members of university faculties who in turn incorporate new developments into *their* advanced graduate courses, and so on. At the same time, and in a lower circle, budding teachers of mathematics, including schoolteachers and those destined to become teachers of schoolteachers, are preparing for their careers. Faced with the necessity of acquiring competence in not just mathematics but in pedagogy as well, and in related sciences that they must often be prepared to teach, they seldom reach the advanced graduate courses and the new ideas before setting out to teach the old ideas to their students, some of whom become teachers who teach the old ideas to *their* students, and so on. While it cannot be denied that there is some seepage from the upper circle to the lower, the time lag is such that the new ideas are decades out of date by the time they reach the undergraduate curriculum, and their approach to the secondary curriculum is essentially asymptotic.

It is interesting to note, by the way, that in sciences other than mathematics some attempts are being made to hasten the seepage from the upper circle to the lower. For example, committees of the American Society for Engineering Education and the American Association of Physics Teachers have recently undertaken studies and conferences on the place of physics in engineering education, including nuclear physics and solid state physics. Again, last summer the Oak Ridge Institute of Nuclear Studies offered a two-week course for high school science teachers on the use of radioisotopes, and a member university of the Institute is offering a similar course this coming summer. Quite apart from questions of merit or effectiveness, these undertakings are important in that they reflect a sober concern for modernizing science instruction at lower levels of the educational structure.

The need for change

In saying that there is need for change in secondary school mathematics, I do not mean need for change in the hackneyed "there is always room for improvement" sense; I mean urgent and critical need for change.

Consider the need, if you will, in terms of natural resources. As a nation we are proud of our resources and our concern for developing and utilizing them wisely and well. And no one will deny that our greatest resource is youth. Yet of the students who graduate from high school and are competent to do college work, five out of ten never get to college, and of the five who do, two never graduate.

Or consider the need, if you prefer, in terms of scientific manpower. The importance of science in the nation's defense and economy, magnified beyond all expectations during and since World War II, has been thoroughly demonstrated and documented. No one will deny the importance of insuring a swelling flow of abler and better educated scientists and engineers.

Or consider the need in terms of secondary school science and mathematics teaching. Since 1950, due primarily to reduced numbers of births during the depression, the number of college graduates has decreased each year. Decreasing at a still greater rate is the number of college graduates prepared to teach in secondary schools. But greatest of all, and most alarming, is the rate of decrease in the number prepared to teach secondary school science and mathematics—and this at a time when the increased birth rate following the depression has resulted in a wave of students already flooding the elementary schools.

I take it as self-evident that, by and large, motivations and decisions for or against college are developed in our youth during their secondary school years. What about motivations and decisions for or

against science or science teaching as a career? While little information is available about career choices of scientists as teachers, one study of distinguished scientists—biographees whose names were starred in *American Men of Science* during the forty years ending in 1943—reveals that some 60 per cent had made the decision for science when eighteen years old or younger.[9] A more recent study of origins of American scientists reveals that the age of decision is dropping. In the case of the physicists included in this study, for example, only about one-quarter of those who received their Ph.D.'s during the years 1918–1924 decided to become scientists before their seventeenth year; whereas, of those who received their Ph.D.'s during the years 1934–1940, over two-thirds made the decision before their seventeenth year.[10]

Whether viewed, therefore, from the standpoint of proper cultivation and utilization of our intellectual resources generally, or from the standpoint of increasing our scientific potential, or from the standpoint of meeting the growing demand for science teachers, the need for a secondary school mathematics curriculum relevant to modern mathematics and science is clear. Our future leaders in all fields, and our future scientists and teachers of science in particular, deserve a truer and more challenging picture of mathematics than they are now getting.

Some hazards

It is of course easier to cry need for change than to describe changes needed and discover how to bring them about. The course to be set, like so many important courses of action, has its Scylla and Charybdis. I am reminded of these sources of danger by two cartoons from *Collier's* that have occupied honored places on my bulletin board.

One of these cartoons appeared fifteen or twenty years ago. It depicts a first-grade scene in which most of the youngsters are busily engaged with sandbox toys and other play equipment. An exception is the little boy in the foreground, his attention fixed on a blackboard on which he has just written $\frac{+2.}{4}$ "2". Over him stands the teacher, saying, "Come, come, Philip, stop fooling around." What evokes our smiles is, of course, our conviction that it is not Philip, but the teacher, who is fooling around. This is one of the dangers we run.

A classic example of fooling around is what has happened to plane geometry during the last couple of decades. In seeking to justify its place in the secondary school curriculum, teachers seem to have settled upon the conviction that it is the best means whereby to teach students to think logically and express themselves precisely. This conviction was presumably responsible for the publication of a textbook entitled, if I am not mistaken, *Clear Thinking, an Approach Through Plane Geometry*. I believe most mathematicians today are agreed that the ends sought are more readily and safely achieved through the study of more transparent mathematical systems such as finite geometries or, even better, certain simple algebraic systems. The logical obscurities and pitfalls of Euclidean plane geometry becomes readily apparent to anyone who tries to struggle through Hilbert's *Foundations of Geometry*,[11] for example, in complete detail.

Fooling around with the conventional curriculum serves only to mislead our colleagues in science and the public at large. There is a growing body of natural scientists who urge that every able high school student be required to take four years of mathematics, even if it means foregoing

[9] S. S. Visher, *Scientists Starred, 1903–1943* (Baltimore: Johns Hopkins Press, 1947), p. 526.

[10] R. H. Knapp and H. B. Goodrich, *Origins of American Scientists* (Chicago: University of Chicago Press, 1952), p. 423.

[11] D. Hilbert, *Foundations of Geometry*, translated by E. J. Townsend (La Salle, Illinois: Open Court Publishing Company, 1902).

science altogether in school. Surely these people assume a four year program of mathematics relevant to modern science. Are they safe in their assumption? In his annual report for 1953–54 Henry Chauncey, President of the Educational Testing Service, speaking of the need for greater educational efficiency, says:

> To provide adequate mathematics training for the ablest students necessarily requires the availability of a full complement of advanced mathematics courses—e.g., solid geometry, spherical trigonometry, etc.—of good quality.[12]

According to MacLane, however:

> No matter how much better [the old] ideas are taught to more and more pupils, their presentation leaves school mathematics in a state far more antiquarian than that of any other part of the curriculum. The pupils can conclude only that there is no such thing as a new mathematical idea.[13]

The second cartoon from *Collier's* that graces my bulletin board depicts a scene in what is perhaps a second-grade classroom. The teacher is at the board, writing down some simple arithmetic exercises. While her back is turned, one bright-looking pint-sized boy is whispering to another, with obvious earnestness, "It isn't the atomic theory that bothers me—it's the adding and subtracting."

The hazard implied by this story has been pointed up as follows by George Harrison, Dean of Science at the Massachusetts Institute of Technology, in an article on secondary school science teaching:

> A great danger in secondary school education is the growth of superficiality. Under the guise of modernity high school teachers embark enthusiastically on programs regarding atomic energy, for example, who have almost no understanding of the laws governing energy in general. . . . While this sort of thing may be justified as helping to motivate students toward science, it is likely to motivate the wrong people.[14]

[12] *Annual Report to the Board of Trustees, 1953–54*, Educational Testing Service, pp. 21, 22.

[13] MacLane, *loc. cit.*, pp. 66, 67.

[14] G. R. Harrison, "The Role of the Secondary School in the Teaching of Science," *Physics Today*, Vol. 5, No. 6 (June, 1952), p. 14.

Our course, then, should be carefully set to avoid on the one hand the sterility that lurks in reworking an old field already too thoroughly worked over, and on the other the superficiality that lurks in too abrupt a conversion to modernity.

There is yet a third hazard to be avoided. I refer to the stubborn habit teachers have of underrating the ability of fourteen to eighteen-year-olds to grasp and use high-level abstractions characteristic of modern mathematics. It has always seemed to me on a priori grounds that young people of that age are far more at home among abstract concepts and phenomena than among the social concepts and phenomena they are asked to learn about before having had the experiences necessary to an understanding of them. There are said to be studies substantiating this view, but I have not been able to locate them. My own conviction is grounded in twenty years of efforts to push youngsters to greater and greater lengths. As evidence of early conviction I cite the fact that at the annual meeting of The National Council of Teachers of Mathematics fourteen years ago I addressed the private school section under the title, "Don't Forget the Good Student!" As evidence that but few of my colleagues share my conviction, I cite a further fact. At the same meeting of the National Council a new geometry text was being defended by one of its authors, Ralph Beatley.[15] Only a few years later it was out of print—teachers had pronounced it too difficult. It was, in fact, simpler and more elegant conceptually than the conventional treatment, and far more up to date in point of view. Too many teachers underestimated the capacities of their students.

Some pointers

But enough of hazards to be avoided in setting the course. Let me speak rather of

[15] G. D. Birkhoff and R. Beatley, *Basic Geometry* (Chicago: Scott, Foresman and Company, 1941).

the course to be set. I should have said "courses," naturally, since the ends to be achieved must be approached along various routes by various groups.

MacLane observes that the isolation of the secondary schools from the intellectual life of modern mathematics is "no one's fault and everyone's negligence."[16] This apparent paradox is resolved by Alden Emery, Executive Secretary of the American Chemical Society, in this way:

> In no field of science does the knowledge which one gains in college remain adequate for any length of time. . . . The research specialist in industrial or academic work can select from the tremendous volume of scientific literature being published, those articles within his narrow field of specialty. The high school teacher cannot do so because his specialty encompasses the entire field of knowledge in one or more scientific disciplines. Most of our institutions of higher learning and our professional societies have not provided a mechanism by which high school science teachers can keep in touch with new developments as a whole.[17]

Both MacLane and Emery suggest what seems to me to be obvious and inescapable: that a secondary school mathematics curriculum relevant to modern mathematics and science is to be achieved not by teachers alone, nor by mathematicians alone, but by teachers and mathematicians together. I conceive of the strategy required as a kind of "Operation Bootstrap," designed to enable those of us who teach at the secondary school level to lift ourselves, together with the lower of the vicious circles I described earlier, to a level high enough to establish contact with the upper circle. Other members of the mathematical community can and must help provide the bootstraps, but the will to lift must come from the owner of the boot.

What are some of the bootstraps that others can provide? A few are already in the making. I have in mind, in the first instance, expository writings by mathematicians about modern mathematics. An example is the paper by MacLane to which I have so frequently referred. It merits wider distribution to teachers than it currently enjoys in the *Bulletin of the National Association of Secondary School Principals*, for it is the most successful attempt I have seen to describe in terms of conventional secondary school course materials some of the concepts and ways of thinking that are typical of modern mathematics. Again, there is the forthcoming National Council *Yearbook* that is to consist of a collection of monographs on modern mathematics written specifically for teachers by several distinguished mathematicians. Nor should we overlook the growing number of new college texts that seek to incorporate modern mathematics in undergraduate courses. Careful individual examination and study of such expository materials can be a proper and effective first step.

Increasing in both number and variety are other means, more concentrated than programs of individual study, by which teachers who seek to inform themselves about modern mathematics and its possibilities for the instruction of modern youth are enabled to do so. I refer to programs of summer instruction, summer institutes, visiting lecturers, teacher fellowships, and the like, supported by industrial corporations, private foundations, and government agencies.

In the general plan of attack, let us not overlook the preparation of new teachers. This responsibility calls for the cooperative efforts of educators and administrators of secondary and higher education, as well as of mathematicians and teachers. I shall not attempt here to describe the mathematical content of a master's program, say, appropriate to tomorrow's mathematics teacher—at this point I dare not stretch my hour and your patience. Let me rather refer you once more to MacLane, this time to his Retiring Presidential Address before the Mathematical Association of America in 1953. Look for

[16] MacLane, *loc. cit.*, p. 67.

[17] A. H. Emery, "The Education Team," *The Science Teacher*, Vol. XXII, No. 1 (February 1955), p. 10.

it in the *American Mathematical Monthly*;[18] you will find it to be still another helpful source of information about the transition from classical to modern mathematics.

Conclusion

In concluding, I earnestly urge that we teachers of mathematics invite other members of the mathematical community to join us at once in the careful planning and prompt launching of some sort of "Operation Bootstrap." The time, it seems to me, is ripe for such an undertaking. For one thing, our colleagues the mathematicians seem readier and more willing to help than for some decades past. For another, public awareness of the critical role of secondary school mathematics in the nation's defense and economy has reached new dimensions, and there is every reason to believe that funds from industry, government, and other sources will be forthcoming in support of wise and effective plans.

Added urgency is provided by a look at enrollment in the elementary schools. The approaching wave of students will soon engulf first the secondary schools and then the colleges, and we shall have time for little but teaching. Let us, while there is still time to think and plan and learn, call upon all of the professional and financial resources available to bend their concerted efforts and influence to the task of helping us to get ourselves and our house in order.

[18] S. MacLane, "Of Course and Courses," *American Mathematical Monthly*, Vol. LXI, No. 3 (March 1954), pp. 151–157.

Mathematics and science contests

If you know of a mathematics contest for secondary school students being conducted in your area, will you please send a postcard to the undersigned, giving the name and address of the responsible person, or organization, whom we should contact for information about it? This will aid us greatly in locating such contests.

The National Council is making a survey of all such contests and will need the cooperation of many members in trying to locate them. Generally the persons conducting such contests are so busy doing it that they do not have time to answer this appeal, unless you help us spot them.

So, before you forget it, do your good deed today—drop a card in the mail, tipping us off. Address it to

Daniel B. Lloyd,
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Wilson Teachers College,
11th & Harvard Streets, N. W.,
Washington 9, D. C.

Where the mathematics teacher is king[1]

PAUL R. NEUREITER, *State University Teachers College, Geneseo, New York.*

This note about teaching in the Netherlands supplies information enabling readers to make some interesting comparisons with the work they are doing in mathematics.

IF AN AMERICAN mathematics teacher wants his ego inflated, he should go to the Netherlands and find himself a job in a secondary school there. Despite a noticeable deflation of his pocketbook, he will bask in a sense of self-importance that he has never known in this country. For mathematics is the kingpin of the curriculum in the Dutch primary and secondary schools, and the teacher of mathematics is the demigod who holds the future fate of many a youngster in his mighty hand.

This came as a surprising experience to the writer when he had occasion to spend a year in the Netherlands under the Fulbright Program, teaching in the public schools and studying the school system of the Netherlands. He experienced the same kind of shock that Professor Fehr writes about in a recent book review of a German secondary school text (THE MATHEMATICS TEACHER, October, 1954) when he says: "The teacher in the United States of America will find the comparison of what is taught in the German *Gymnasium* with the typical American program a real shock . . . " But Professor Fehr softens the blow by reminding his reader that the German *Gymnasium* caters, after all, only to the topmost 10 per cent in intelligence and scholarship.

The same thing is certainly true of the secondary schools in the Netherlands. But the writer found that the exalted position of mathematics in the schools for the select minority contributes to a very high regard for mathematics in the other kinds of schools, so that in all branches of the highly diversified and specialized system a considerable proficiency in mathematical operations is required of every youngster who wants to earn any kind of school diploma and aims to qualify for a job outside the unskilled labor category.

While the most difficult type of secondary school graduates only 9 per cent of the school population, there is another kind of high school that educates about 25 per cent of the young people between the ages of twelve and seventeen. In their aversion to inflated currency of any sort the Dutch call it "advanced elementary school," but its course of study requires three or four years of mathematics along with three foreign languages. There is a similar mathematical influence in the curricula of the agricultural, home economics, and trade schools, and all of this makes in turn for an intensive cultivation of arithmetic in the grades.

The very nature of the Dutch curricula favors the highly sequential subjects such as mathematics and the foreign languages. While there is freedom in the choice of a secondary school curriculum and the sys-

[1] Reprinted from *New York State Mathematics Teachers' Journal*, Vol. 5, No. 2 (April 1955), pp. 6–9.

tem offers an astounding variety of them, there is no election of subjects after the choice is made. The pupil is caught in a stream of between 12 and 20 required subjects, most of which continue for four, five, or six years. He has mathematics in every grade and grows up with it. No one would say, as with us, that he "is taking a year of algebra," or "a year of geometry." Both disciplines are taught side by side for years, and later trigonometry, solid geometry, and sometimes analytics are added to the systematic course of study, which emphasizes prolonged and steady exposure at the rate of one or two lessons per week in each branch of mathematics. The almost perpetually dripping sky of the Low Countries encourages the notion that a continuous drizzle is better than an occasional cloudburst.

The American teacher who identifies himself temporarily with a foreign school system becomes cognizant of strong and weak points on either side. In Holland he cannot help but admire the tremendous capacity for work, the industry and perseverance of the students; but he misses the friendly, free-and-easy cooperation of pupils and teachers, so typical of our schools. And he regrets the absence of a youthful zest among the students, who by nature are just as capable of it as our pupils but are not encouraged to develop it.

In the mathematics classroom this means specifically that a small minority of youths with a high aptitude for mathematics are really enthusiastic about it, whereas the majority are either indifferent or even hostile to it. So far as insight and understanding are concerned, they do not seem to be as widespread as one would expect in view of the time and effort devoted to the subject.

The writer noticed a high degree of standardization in the methods of instruction, probably the result of the central control exercised by the Ministry of Education. He met fewer examples of brilliant performance among the students than he would have expected in view of the elite character of the student body. As a matter of fact, he observed a surprisingly large number of students who were plodding through advanced areas without any real talent or liking for mathematics; and he was told time and again by disgruntled students that "much of what we learn will be useless to us in later life." This is undoubtedly so because many graduates do not enter technical professions related to mathematics. They study the subject solely because it is required for getting a secondary school diploma, which is considered socially advantageous as a pass for admission to higher-paying jobs.

The golden fruits of Dutch mathematics teaching are most tangible in the case of the brilliant student who enters a higher institution of learning (which is practically a graduate school). There the advantages of the European system are undisputed. The university does not have to struggle with poorly prepared students. University standards do not have to be diluted to accommodate the deserving but ill-prepared neophyte. Students are able to begin professional programs fully qualified at an earlier age than here, a point of economic importance in these times of compulsory military training and long years of preparation for the professions.

Nothing could give as clear a picture of the caliber of mathematics taught in the Dutch schools as a sampling of problems that have actually been given on final examinations in recent years. The Dutch are fond of comprehensive finals in almost all types of schools, and mathematics is rarely excluded from them. So here are some questions that the author gleaned from official examination records. You may wish to try these on your students.

First, two examples that appeared on the entrance examination that a youngster, twelve or thirteen years old, has to pass after sixth grade to gain admission to the secondary school of the highest type. The problems must be solved without the use of algebra.

1. Three sums are in the ratio 5:8:11. They are invested at 3 per cent per year. The middle sum yields 800 guilders in 8 months. What are the three sums?

2. Three persons, A, B, and C, buy postage stamps. A buys stamps at 10¢ apiece, B at 6¢ apiece, and C at 2¢ apiece. A buys twice as many stamps at 10¢ as B at 6¢, and B buys twice as many stamps at 6¢ as C at 2¢. Since B spends 50¢ more than C, how much does each one have to pay for his stamps?

Next, an illustration from the comprehensive finals in the "advanced elementary school" (U.L.O. School) in 1952. The age of the examinees is 16 or 17 and the curricula permit a choice between a longer and shorter course in mathematics. The easier algebra test reveals these questions:

1. $\dfrac{\sqrt{4-\sqrt{7}}+7\sqrt{1/14}+3\sqrt[4]{36}\div\sqrt{12}}{\sqrt{14}-\sqrt{2}}=$

2. $\dfrac{(a^{p+2})^{p-3}\times(a^{2p+2})^{2}}{(a^{p+1})^{p}\times\sqrt[3]{a^{3p}}}=$

The more difficult algebra test in the same kind of "advanced elementary school" has these questions on it:

1. An arithmetic and a geometric progression each have three terms; the first terms of the two progressions are identical while the sum of the terms in each progression is 28. It is also given that the difference in the arithmetic progression is 6 times the first term. Determine the two progressions.

2. When one inverts the roots of the equation $8x^2-2ax+a=0$, one obtains the roots of the equation $9y^2-2by+b=1$. Determine a and b.

The geometry test of the same final contains this problem: In a circle with center at M the chord AB is drawn. The midpoint of AB is N. The tangent to the circle in B intersects MN extended in S. Through S an arbitrary secant is drawn through the circle. Its points of intersection with the circle are C and D. Prove: $SN\times SM=SC\times SD$.

Finally some examples from the comprehensive final in the Hogere Burgerschool, the secondary school emphasizing mathematics and the exact sciences, whose successful graduates are automatically admitted to the Institute of Technology at Delft.

In algebra: 1. Approximate the root of the equation $3^{2^x}=2^{3^x}$ with the aid of a logarithm table.

2. a. The graph of the function

$$y=\frac{ax+b}{x+c}$$

has asymptotes whose equations are $x=3$ and $y=2$. The graph goes through the point (1, 3). Determine the function and sketch the graph.

b. If P and Q are the points of intersection of the graph with the coordinate axes, determine the equation of the straight line PQ.

In trigonometry: 1. In a circle with radius R two triangles are inscribed. In the first triangle the sides are in the ratio 3:4:5, in the second triangle the angles are in the ratio 3:4:5. Determine the number that indicates how many times the area of the first triangle is contained in the area of the second triangle, if necessary, with the aid of a table of logarithms.

2. a. Solve:
$10\sin^2 x-24\sin x\cos x=p$, when $p=-3$.

b. For which values of p does the above trigonometric equation have no solution?

Can our freshmen in engineering schools do these problems?

Where do eligible mathematics teachers go?

RAY C. MAUL, *Research Division, National Education Association, Washington, D. C.*

A timely report on a most pressing problem. Teachers can help by publicizing the facts presented in this article.

WE KNOW, of course, that the shortage of competent elementary school teachers has been critical for a number of years. We know, also, that the former rather comfortable situation at the high school level is now in the early stages of what will inevitably be a profound change. Only a few years ago it was assumed that there were enough qualified candidates—at least in most of the subject fields—to meet the demands for replacement of high school teachers who quit the classroom, but now everybody knows this is no longer true.

A timely question, then, is this: What happens, by way of occupation, to the students who graduate, year by year, from the colleges with preparation to enter the teaching profession? Even more to the point: What happens to the college graduates with major preparation in mathematics who meet the requirements for the teaching certificate? Do they actually seek teaching jobs? If not, what do they do?

The picture becomes a little clearer as the contents of the Eighth Annual National Teacher Supply and Demand Report[1] are examined. This report shows that the number of college graduates prepared for elementary school teaching, while far too few to meet the needs, has inched up a little, year by year, since 1950, but the number of graduates prepared to become high school teachers has fallen off sharply, and the number prepared to teach mathematics has decreased at a more rapid rate. The evidence indicates that high schools are at the threshold of an entirely new era.

Three important factors claim particular attention as the over-all situation is examined.

First, America is in greater need of trained scientists today than ever before. As international tensions increase, our continued leadership is challenged in many ways, but more obviously in the maintenance of our technological superiority. This has been the most distinctive characteristic of the rise of America as a world power. If it is to be preserved, the *high schools* must seek out, identify, and encourage those students who have the innate capacity for advanced scientific training; the *high schools* must kindle the interest, fire the imagination, and inspire these young people to continue into college for advanced training (some to become teachers, but many to enter and carry forward every kind of scientific endeavor). This is peculiarly the job of the high schools, and more especially of the teachers in the scientific fields. This unending search for talent is and must be

[1] The study is conducted by the NEA Research Division; the report is published by the NEA Commission on Teacher Education and Professional Standards in the March issue of the *Journal of Teacher Education*.

based on qualitative standards, and high school teachers are best situated to exercise objective judgment.

Second, the high schools must prepare to serve the unprecedented number of students who are soon to attain high school age. The vanguard of this avalanche is just now touching the seventh and eighth grades; all of the impact upon grades 9–12 is yet to be felt. And to complicate the matter further, this vast increase in total numbers to be educated will not strike local communities or states in a uniform manner or at a constant rate. The increase will be distributed unevenly, from school district to school district, and will tend to shift in response to changing economic factors.

Third, in addition to the task of admitting greater numbers, the high schools must re-examine their effectiveness in holding all students by meeting their varying needs. Only recently have the high schools of the nation succeeded in holding more than half of the entering freshmen through to graduation. This expanding *quantitative* task stands out more and more as the American people gain in their appreciation of the value of education. High school administrators and teachers must reconcile their efforts here with their efforts to strengthen the *selective* encouragement of their most capable students.

These three factors—just now being recognized on the American scene—lead directly to the quest for more teachers, many more competent, fully qualified teachers of all subjects. But these factors point particularly to the need for mathematics teachers, since the study of this basic science undergirds our progress in all scientific and technological fields. Thus we face the question: Where do mathematics teachers come from?

Since college graduation is a minimum requirement, all new teachers must come from the current college graduating classes or from the classes of earlier years. Qualified graduates of earlier years who did not enter or remain in teaching are numerous, but the number who will change their minds and seek teaching at the beginning of any school year is most uncertain and cannot be determined in advance. This means, then, that the only measurable source of supply of new teachers is the current 1955 graduating class.

Table 1 shows that this annual source of new supply of eligible candidates has been in a steady downtrend for the past five years. In 1950 the total number of college graduates (Bachelor's degree) reached an all-time high—almost 434,000. Of this number almost 87,000 met the requirements (averaging 18 semester hours of professional courses ordinarily taken during the junior and senior years) for the high school teaching certificate. Of this latter number 4,600 were mathematics majors. Five years later, in 1955, the graduating class totaled only 284,000, a decrease of 34.4 per cent. In the 1955 class there were only 51,400 graduates prepared to teach in high school, a decrease of 40.8 per cent. And in this latter group there were only 2,250 mathematics majors.

This is a 51.3 per cent drop in five years in the number of college graduates fully prepared to teach this vitally important subject.[2]

The diminishing supply of eligible qualified candidates to teach mathematics sharpens our interest in the question: Where do eligible mathematics teachers go? Is it safe to assume that all of the 2,250 qualified mathematics majors in the class of 1955 actually sought and entered teaching positions? And if not all of them, how many became teachers, and how are the others occupied? This information is not yet available, but the national report mentioned earlier shows what happened to members of the class of 1954 according to their occupations on November 1 of that year. These facts are shown in Table 2.

[2] Every college class contains numerous graduates who have made *minor* preparation in mathematics, but since the *major* interests of these graduates are in other fields they will naturally seek to teach these other subjects if they enter teaching.

TABLE 1

TOTAL NUMBER OF COLLEGE GRADUATES; NUMBER PREPARED TO TEACH THE VARIOUS HIGH SCHOOL SUBJECTS; NUMBER OF MAJORS IN MATHEMATICS WHO PREPARED TO TEACH IN HIGH SCHOOL; PER CENT OF DECREASE SINCE 1950

College graduates	1950	1951	1952	1953	1954	1955
Receiving Bachelor's degrees	433,734	384,352	331,924	304,857	292,880	284,500
Per cent change from 1950	—	−11.4%	−23.5%	−29.7%	−32.5%	−34.4%
Prepared to teach various high school subjects	86,890	73,015	61,510	54,013	48,916	51,418
Per cent change from 1950	—	−16.0%	−29.2%	−37.8%	−43.7%	−40.8%
Majors in mathematics prepared to teach	4,618	4,118	3,142	2,573	2,223	2,250
Per cent change from 1950	—	−10.8%	−32.0%	−44.3%	−51.9%	−51.3%

TABLE 2

OCCUPATIONS, ON NOVEMBER 1, 1954, OF MEMBERS OF THE 1954 GRADUATING CLASS WHO PREPARED TO TEACH; PER CENT ENGAGED IN TEACHING AND IN OTHER ACTIVITIES

Field of preparation	Per cent of total							
	Teaching	Otherwise gainfully employed	Continuing formal study	In military service	Homemaking	Seeking employment	No information	Total
Various high school subjects:								
Men	47.5	7.6	8.5	19.7	0.0	2.2	14.5	100.0
Women	64.8	7.8	4.3	0.2	7.3	2.7	12.9	100.0
Total	55.7	7.7	6.5	10.5	3.4	2.5	13.7	100.0
Mathematics:								
Men	52.9	8.5	7.9	24.4	0.0	0.8	5.5	100.0
Women	69.3	11.5	3.7	0.3	7.7	1.2	6.3	100.0
Total	59.0	9.6	6.3	15.5	2.9	0.9	5.8	100.0

As shown in Table 1, the total graduating class in 1954 numbered 292,800, of whom 48,916 prepared to teach in high school, of whom 2,223 were mathematics majors. Table 2 shows that, of the 48,916, only 55.7 per cent actually entered teaching; of the 2,223 mathematics majors, only 59.0 per cent did so. (When these 2,223 mathematics majors are divided by sex, Table 2 shows that 52.9 per cent of the men and 69.3 per cent of the women took teaching positions.) In other words, 4 of every 10 members of the 1954 class who were qualified to become teachers of mathematics did not do so. And to gain the full meaning of this loss of eligible candidates, it must be repeated that this 1954 class of 2,223 eligibles was less than half the size of the 1950 class of eligible candidates for mathematics teaching positions.

But what happened to that 41 per cent of the 1954 class that did not enter teaching? Table 2 shows that 9.6 per cent are otherwise gainfully employed, 6.3 per cent are continuing formal study beyond the Bachelor's degree, 15.5 per cent are in military service, 2.9 per cent devote full

time to homemaking, 0.9 per cent are seeking employment, and 5.8 per cent are unaccounted for. As is always the case with college graduates eligible to enter teaching but who do not do so as they undertake occupational life, some of this "lost" supply may later become available. The hard fact remains, however, that school employment officials are able to attract only a few more than one-half the college graduates who showed sufficient interest in teaching to take the courses required to obtain the certificate.

The facts presented in the national report are cause for grave concern. Teachers of mathematics are in a strong position to interpret these facts to citizens in the local communities. If the unfavorable trend is to be checked—if more of our capable high school graduates are to enter college, if more of them are to elect teacher-education programs, and if more college graduates are to enter teaching—the problem of improving the competitive attractiveness of teaching must be vigorously and courageously attacked in every local school district. Teachers surely can (a) advise civic groups and local leaders of the changing conditions described above, and (b) point out to promising young students the many rewards and satisfactions to be achieved in teaching. All citizens can take part in strengthening the support of public schools.

But the first step in any such constructive movement is to get the facts. The intent of the annual investigation of the NEA Research Division is to assemble and disseminate such facts as will make for fuller understanding.

What's new?

ERRATUM

The Japanese Abacus: Its Use And Theory, by Takashi Kojima, Tokyo, Japan, and Rutland, Vermont, Charles E. Tuttle Company, 1954. 102 pp., $1.25.

BOOKS

Secondary

Algebra One, Rolland R. Smith and Francis G. Lankford, Jr., New York, World Book Company, 1955. Cloth, vi+410 pp., $2.80.

Algebra Two, Rolland R. Smith and Francis G. Lankford, Jr., New York, World Book Company, 1955. Cloth, vi+506 pp., $3.00.

Functional Mathematics, Grade 7, William A. Gager, Beulah Echols, Carl N. Shuster, Richard Madden, Franklin W. Kokomoor, New York, Charles Scribner's Sons, 1955. Cloth, viii+390 pp., $2.24.

Functional Mathematics, Grade 8, William A. Gager, Dorris H. Johnson, Carl N. Shuster, Richard Madden, Franklin W. Kokomoor, New York, Charles Scribner's Sons, 1955. Cloth, viii+373 pp., $2.24.

New Solid Geometry, A. M. Welchons and W. R. Krickenberger, Boston, Massachusetts, Ginn and Company, 1936, 1943, 1950, 1955. Cloth, x+326 pp., $2.68.

One Hundred Mathematical Curiosities, William R. Ransom, Portland, Maine, J. Weston Walch, Publisher, 1955. Paper, vii+212 pp., $3.00.

College

Advanced Calculus, Angus E. Taylor, Boston, Massachusetts, Ginn and Company, 1955. Cloth, xiii+786 pp., $8.50.

Algebraic Geometry, Clement V. Durell, London, England, G. Bell and Sons, Ltd., 1955. Cloth, xvi+387 pp., 18*s* 6*d*.

Almost Periodic Functions, A. S. Besicovitch, New York, Dover Publications, Inc., 1954. xiii+180 pp., Paper-bound: $1.75, Cloth-bound: $3.50.

Analytic Geometry, Neal H. McCoy, and Richard E. Johnson, New York, Rinehart and Company, 1955. Cloth, xiv+301 pp., $4.00.

Practical mathematics is challenging to students[1]

FRANCIS G. LANKFORD, JR., *University of Virginia, Charlottesville, Virginia.*

Many students like to know that the mathematics they must study can be used; however, practical problems must be wisely selected to be of interest to students.

As I EXAMINED the title assigned me for this speech I could not at first decide whether to consider it an assumption and proceed to examine its reasonableness or to consider it a conclusion—a theorem proved—and proceed to give a number of examples or implications of the conclusion. I guess what I have planned to say fits better under an interpretation of the subject as an assumption. Before proceeding with my examination of this assumption, though, there is a point which I want to make very clear at the outset. It is simply this. To assume that "If mathematics is practical, it is challenging to students" is not necessarily to assume the converse, namely, that "If mathematics challenges students, it is practical." I would not wish to defend this converse. In fact I am certain that using applications in teaching mathematics to students is only one means of challenging students. We have all taught mathematics to students who never ask, "What good is this going to do me?" We have doubtless had many others who have persistently raised this annoying question. I distinctly remember an experience of several years ago when I was teaching algebra in a school where algebra was required for graduation. A girl interrupted my teaching with the question, "What good is this stuff going to be for a nurse? I am going into nursing if I ever get through this school." The difference seems to be explained by the fact that many pupils who understand the principles and methods of mathematics are stimulated by its systematic and logical nature even though at times highly abstract.

Before we examine the assumption that practical mathematics is challenging to students maybe we had better be agreed on what practical mathematics is. If you will reflect on this definition a moment, you will soon see it is not immediately obvious. Are we to say that any mathematics is practical that may be used? With this broad definition, we would have to say, for example, that quadratic equations are a very practical bit of mathematics content. Similarly most of the content of plane and solid geometry as well as of trigonometry would have to be judged practical. Here, you see, we are in the very troublesome dilemma of trying to distinguish between what is pure mathematics and what is practical. I am afraid we could spend the rest of the time I have allotted to me in trying to make this distinction clear and mutually exclusive. Instead of engaging in this academic and thoroughly impractical exercise for our purposes here, I will attempt a simple and working definition by saying that "practical mathematics is applied mathematics." When mathematics is used it becomes practical mathematics. In accord with this idea, we may say that using case I of per

[1] Delivered before the Thirty-second Annual Convention of The National Council of Teachers of Mathematics at Cincinnati, Ohio, April, 1954.

cents to find how much income tax a person must pay on his declared net income is an example of practical mathematics. Similarly a carpenter's use of the Pythagorean theorem to determine the length of rafters for a house or stringers for basement steps; using addition and subtraction of decimal fractions to keep one's bank account on the stubs of his checkbook; using trigonometric ratios to locate the boundaries of a city lot; or a farmer's use of ratio in regulating the feeding of a dairy cow are all examples of practical mathematics. Some of you may want to disagree with this definition I have tried to give for practical mathematics. Such disagreement would not disturb me. I have merely tried to set forth the interpretation I have used in the following.

I should like to proceed now to an examination of some of the conditions under which practical mathematics is challenging to students. The *first* condition I would mention is that *the applications used should be real ones.* This condition means that a teacher will frequently supplement the textbook treatment of an application with live data or information collected by the students. Let us imagine that a teacher is using operations with large whole numbers in a practical study of the characteristics of our population. She may very well start with an analysis found in her textbook and then stimulate her pupils to consider other characteristics that may be particularly pertinent to the locality or age group represented: This will of course involve consulting such library sources as *The World Almanac* or the annual *Statistical Abstract of the U.S.* or various census reports. Not always will sources of this kind be turned to for supplementary information. A class may be studying the relative cost of renting or buying shelter. Again the generalized conditions treated in the textbook may become the springboard for a study of the question with local and current data. In this instance students will read newspaper advertisements, and consult real estate dealers—many of whom, incidentally, will be their parents—as sources of the information they need. When pupils have learned how to make scale drawings through a study of how to choose and use appropriate scales, they may then have their interest considerably stimulated by a study of samples of blueprints of actual houses represented in the class—even the plans for the school building itself. Some students may draw to scale the floor plan of their own room and plan a different arrangement of furniture. Indeed some teachers have reported great student interest in making a floor plan for a whole building, a model of which is actually built in the shop of the school by boys who are taking both shop and mathematics.

A *second* condition under which practical mathematics is challenging to students may be stated like this. *The applications must be within the reasonable comprehension of students.* There are those who maintain that so-called pure mathematics is simpler and easier to learn than applied mathematics. And this is the main point of their argument. They believe that many of the applications of mathematics arise out of situations which high school students have no basis for understanding. This is particularly obvious in the case of many of the applications of algebra. Such applications frequently come from the research laboratory or from the technical work of the engineer. To try to use such applications in first course high school algebra, for example, is to confuse most of our students more than to stimulate them. It takes more time and effort of the teacher to explain the setting of the application and therefore its meaning, than the values derived justify. It does not help much for a student to know that algebra is used by the engineer and scientist if he must say, "I don't understand how it is used." It seems to me we must recognize the significance of this point even though some teachers and pupils would like to use more practical verbal problems than the usual "age" problems, "abstract number" prob-

lems, and the "up-and-down-stream" problems. We can substitute jet airplanes for rowboats but we still have pretty artificial problem material. Surely, long before this, such problems would have largely disappeared from algebra courses if equally easily understood real applications could have been found as substitutes. There seems to me one possibility for resolving this difficulty to some extent. In the first place a teacher of algebra, for example, does not need to have ready acquaintance with many of the unusual and perhaps technical applications of her subject. Yet she may profitably encourage individual pupils who have special interests in particular kinds of applications to pursue that interest independently of the rest of the class. Of great help in such individual projects will be a library of applied mathematics books. Many teachers are not aware that there are books with such titles as *Carpentry Mathematics, Sheet Metal Mathematics, Agriculture Mathematics, Aviation Mathematics, Arithmetic for Nurses, Homemaking Mathematics,* as well as *Shop Mathematics* and *Business Mathematics.*

A *third* condition under which practical mathematics is challenging to students is that some *regard must be given to maturity and interests of students.* Recently I was working with a group of junior high school mathematics teachers in our state. One teacher observed during a group discussion, "I think budgeting is not a very practical topic with junior high school pupils. They are too young to care much how family finances are handled." To this another teacher quickly replied, "I don't give much attention to family budgets in my classes. Rather I emphasize personal budgets and budgets of school activities. Often the officers of a school club are in my class. They will supply data from which the class may examine or even plan a budget of a particular club." "Moreover," this teacher continued, "many of my pupils have allowances or earn money in after-school and Saturday jobs. They work with great interest on budgets that help them plan their own personal expenditures."

At another time I visited a class in one of the schools in Washington, D. C. There I saw a group of students applying arithmetic to the study of consumer problems. The class was working on the problem, "Can a young couple afford to get married on $50 a week?" Before I visited the class they had decided how much of this weekly income would have to go for shelter and how much for food. The day I visited the class they were deciding how much would have to go for utilities—gas, electricity, telephone, and water. Each student reported the amount spent for these utilities in his home the preceding month. They then decided how much the young couple would need for these utilities. There was obvious and genuine enthusiasm in this class as they studied budgeting. There are probably several explanations of why this teacher was doing such a successful job. A likely one is that these pupils were now mature enough to be genuinely interested in the problem under study. Indeed some of these pupils were to be married soon after graduation.

Of course this raises the question of how much we can afford to use applications that are practical for the adult and still make them stimulating to students. We must recognize that if we took out of the high school curriculum, in all courses, the content that is not practical here and now in the lives of our students, we would discard a very large part of our present offering, even much of our so-called practical subjects such as homemaking and business education. Therefore, it seems that whether or not an application that comes from adult life may be stimulating to students depends in large measure on how it is presented. Consider the topic of investments so frequently taught in general mathematics courses. This can be taught with very little meaning to students. On the other hand, considerable attention may be

given to savings bonds, Postal Savings, and municipal bond issues—applications that are within the present experiences of our students. Moreover, a visit to a local stock and bond broker's office will help to make problems involving brokerage fees and bond yields seem much more realistic. I have also had good luck in stimulating interest in stocks on the part of junior high school students by letting each one start with an imagined two or three thousand dollars to invest. They consult stock guides and daily newspapers to decide what stocks to buy. They compute costs including brokerage fees and taxes. They read the stock quotations daily for several days. At the end of the period they must sell their entire holdings and compute profit or loss.

There is a *fourth* condition to the use of practical mathematics. Even when applications are taught as they are used in the field from which the application comes, *the mathematics involved should be clearly indicated.* For example, a carpenter uses two steel squares to mark off the saw cuts for the rafters of a roof. Doubtless he is completely unaware that it is the principle of similar triangles that makes this possible. We have, therefore, not exploited the teaching values of such an application until pupils have learned both how to use it *and* the mathematics that underlies it.

Consider this rule which I found in the back of one of the memorandum booklets given to farmers by fertilizer companies: "To find shingles required in a roof: Double the rafter and multiply by length of building. Then multiply by 9 if shingles are exposed 4 inches, by 8 if exposed $4\frac{1}{2}$ inches, and by $7\frac{1}{2}$ if exposed 5 inches to the weather." Now pupils should be helped to see that here the assumption is made that the shingles are 4 inches wide, for 9×4 inches $\times4$ inches $=144$ square inches. Likewise $8\times4\frac{1}{2}''\times4''=144$ square inches and $7\frac{1}{2}\times5''\times4''=144$ square inches. Also doubling the rafter merely gives the area of two rectangles at one operation.

Another such "practical" application which I found is suggested for the mathematically illiterate yet busy businessman. "To find interest for any number of days, multiply principal by number of days, then divide by 120 if interest rate is 3%; by 90 if interest rate is 4%." This "practical" rule is really not complete until we add, "Finally point off two decimal places." It should be very stimulating to pupils to examine the assumption here which is that 360 days constitute a year; then to see how this "practical" rule is an application of the $I=Prt$ interest formula, e.g., $P=\$1{,}000$, $r=3\%$, $t=60$ days.

$$I=\$1000\times\frac{60}{360}\times.03$$

There is a *fifth* condition which is really an implication of some of the others, already mentioned. This is that *we need equipment and supplementary references in order to use many applications of mathematics* to stimulate students. I have mentioned steel squares and newspapers. Many teachers have learned that interest in a banking unit is greater when actual checks, deposit slips, and promissory notes are used. Indeed one of my students this summer told me a few days ago that one of the large banks in her city had so many requests from the schools for samples of the papers they use that they have prepared individual pupil kits which they are glad to provide upon request. When pupils are studying precision measurement in the shop, it is not absolutely necessary to have a micrometer in the classroom but it surely does add to pupil interest and understanding. We help to stimulate interest in indirect measurements when pupils make such measurements with data they accumulate themselves. For this purpose some equipment is needed: a field protractor, or a homemade transit, and measuring tape. One of the teachers in our state stimulated a lot of interest on the part of his pupils in making some of the old measuring instruments described in the yearbook of The National Council of Teachers of Mathe-

matics on "Surveying Instruments." He had his pupils make a particular measurement indirectly with a hypsometer, checking it with a field protractor and tape.

A *sixth* condition for making practical mathematics stimulating is that our *evaluation of the learning of our pupils must test their ability to solve some applied problems.* We all know that the objectives that really function in our teaching are the ones that are expressed in our tests and examinations. We may emphasize practical applications in our teaching and I am sure they will go a long way toward motivating interest. If, though, we do not use on our tests and examinations some practical applications which have not been used in our teaching we soon will find in our students much less concern about applied problems—especially if it takes some little effort to solve them. I realize that this view is close to saying that tests are strong motivators. Some have condemned this view, feeling that it means only teaching for tests. My feeling is that it is perfectly all right to use tests as motivators to learning, so long as they are good and complete tests and motivate the learning we want. Some years ago I did a study of high school education in Virginia. Among other things, I asked employers of high school graduates what their reactions were to the training their employees received in high school. I received many letters from the executives of large and small corporations in Virginia. Among them was a letter from the president of a large textile manufacturing company. He wrote, "There are thousands of high school graduates, particularly girls, who cannot compute a 22½% discount on a $24.95 dress. There is not one in fifty male high school graduates who can work this problem: If a necktie costs $1.25 and a profit of 37 per cent is desired on the selling price, what is the selling price?" I am sure we might not agree with this man completely. Yet my contention here is that we need to save some of the *real* problems of everyday life, which require uses of the mathematics we teach, for our tests and examinations so that we will *know* what our pupils can do.

I want now to conclude this paper by suggesting some sources of the practical mathematics we have been talking about which the teacher may use. In the first place, school life and activities are a valuable source. For example, many teachers have used such school activities as the operation of the lunch room, keeping club accounts, designing shop projects, locating and laying out play spaces as the school playground, keeping attendance records, examining insurance carried by the school, or conducting surveys of the student body. Incidentally, regarding this last illustration here is an opportunity for some practical uses of simple statistics. Many high school teachers have come to recognize that simple statistics constitute a useful branch of applied mathematics which stimulates their students. Does the principal or guidance counselor want to know the vocational interests of the pupils in his school? Why not turn this problem over to the senior group in practical mathematics to solve? It may lead them into questions of sampling and other simple statistical concepts.

Other subjects in the school are valuable sources of practical applications. Measuring lumber in the shop, linear and capacity measurements in homemaking, graphical representation and index numbers in social studies, and using proportions in science are only a few of the many possibilities. Of course, to exploit this source means being familiar with these other fields or having a close working relationship with teachers in these fields.

Contemporary living is a final source of practical applications we may use in our classes. I have already mentioned some of these: characteristics of the population; scientific developments; trends in employment, savings and credit; sports records; and costs of living are some excellent examples of practical applications of arithmetic that we can use with profit.

Letters to the editor

Dear Editor:

The following are some axioms which I would accept as fundamental in determining a policy for textbook reviews in THE MATHEMATICS TEACHER. How would you and your readers feel about them?

1. A major function (perhaps the sole function) of THE MATHEMATICS TEACHER is to serve as a principal tool of The National Council of Teachers of Mathematics in working for the improvement of mathematics education as an important segment of (or factor in) all education.

2. One of the most important factors in determining the content and even, in part, the nature of mathematics instruction is the textbook.

3. Textbook selection is a difficult and crucial task for classroom teachers and/or school administrators.

4. The preparation of improved textbooks is an extremely slow process and one which is very difficult to accelerate.

To me statements 1 and 2 imply that THE MATHEMATICS TEACHER should concern itself with problems 3 and 4. This then leads to:

5. Probably the best way for THE MATHEMATICS TEACHER to assist with problem 3 is via good, helpful reviews of new texts. A continuing policy of publishing such reviews may even help with problem 4.

I see little likelihood that many persons will disagree with 5, but the words "good," and "helpful" rather beg the question there. I also believe:

6. "Good," "helpful" reviews are sufficiently long and detailed to indicate the criteria for the reviewer's judgments and the bases for his criticisms.

7. "Critiques" need not be adverse, but neither should they be suppressed because they are. In fact, the too laudatory or solely laudatory review is usually of little real help with either problem 3 or problem 4. If it is also very brief and general, it probably is of no more value than a bibliographical notice.

8. If reviews are not to be solely laudatory, and if reviewers are to express their honest and sincere judgments, then differences of opinion will inevitably arise. These differences certainly should be recognized and publicized in THE MATHEMATICS TEACHER.

It seems axiomatic to me, further, that all reviews, but especially longer and more critical reviews, should be written by competent, honest, and sincere persons. However, even with a well qualified and sincere reviewer it is inevitable and even desirable that honest differences of opinions among individuals will continue to exist. No "corrections" or retractions of *opinions* should be expected or required, but THE MATHEMATICS TEACHER can and should encourage and print discussions of them. Of course, there must be some limitation on the space devoted to discussions of any one book, but not on the immediate and complete correction of any *errors* or *misstatements* which might occur in a review, in spite of all the efforts of both the reviewers and the editor.

As you see from this, I approve of what I understand to be your policy on reviews; but I do think your readers, as well as authors and publishers, should be aware of this policy and should be explicitly encouraged to enter into the discussions which it should engender.

Sincerely yours,
Phillip S. Jones
University of Michigan
Ann Arbor, Michigan

Dear Editor:

I am indebted to Mr. Chester F. Protheroe for a correction in the tension formula mentioned in the discussion of musical tuning in the February issue under the title "Sebastian and the Wolf." The correct statement is that when we *quadruple* the tension on a musical string, we double the vibration frequency, and thus raise the tone one octave.

Those interested in pursuing the wolf further should consult a physics textbook (the section on sound), and *Webster's New International Dictionary* under "scale," "temperament," and "tone." See also Sir James Jeans, *Science and Music* (Macmillan, 1938) for a complete and interesting treatment of the mathematics of music.

Theodore C. Ridout
Ginn and Company
Boston, Massachusetts

Gentlemen:

May I suggest that the welfare of mathematics is a *world* problem as well as a national one and that the very nature of mathematics, as a veritable world "language" should, and does, lift our horizon that far.

It would please me if our excellent and constantly improving periodical could inform us of interesting practices as they are developing overseas. They may be better than ours or far, far worse. In any case, we should know about them, and current Fulbright scholars might be suitable sources of information.

Respectfully submitted,
Elizabeth M. Cooper
201 East 71st Street
New York 21, New York

• DEVICES FOR THE MATHEMATICS CLASSROOM

Edited by Emil Berger, Monroe High School, St. Paul, Minnesota

Parabola device

by Hugh Olmsted, University High School, University of Minnesota, Minneapolis, Minnesota

This device consists of a pair of crossed wooden sticks joined together at their intersection with a single screw. Thus either stick may be rotated toward or away from the other. Along each stick short brads should be driven at equal distances, and rubber bands stretched between those having alternate positions on the sides of each of the angles. The construction is the same for all four quadrants (Fig. 1). The envelope of the family of straight lines represented by the rubber bands is a parabola, which changes scales as the ticks are rotated about the screw.

To show that the envelope of the family of straight lines represented by the rubber bands between any pair of arms is always a parabola (regardless of the position of the cross sticks) we break the proof into two parts:

Part I. In case the two sticks are perpendicular, they may be placed on the coordinate axes of a rectangular coordinate system where, if the unit distance is conveniently chosen, the equation of the family of lines has the form

$$f(x, y, \alpha) = \frac{x}{\alpha} + \frac{y}{1-\alpha} - 1 = 0.$$

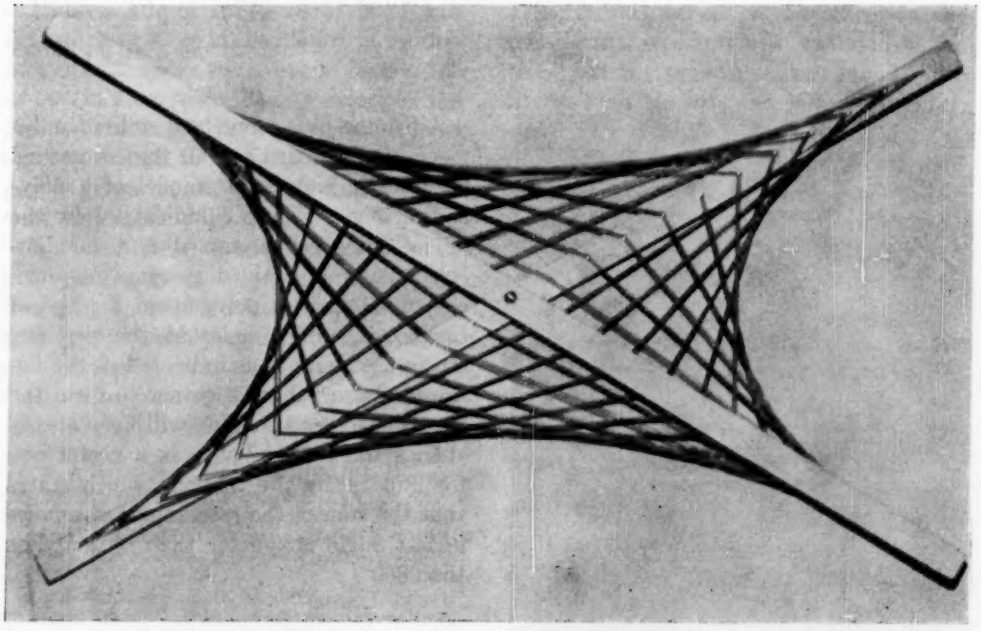


Figure 1

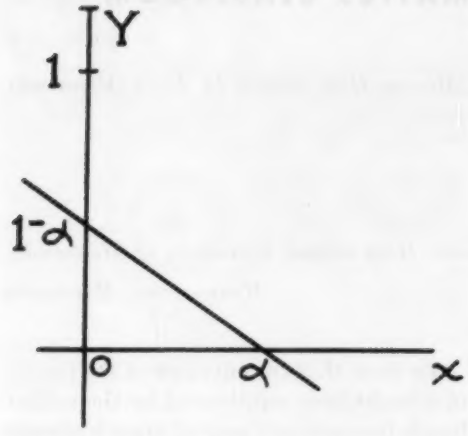


Figure 2

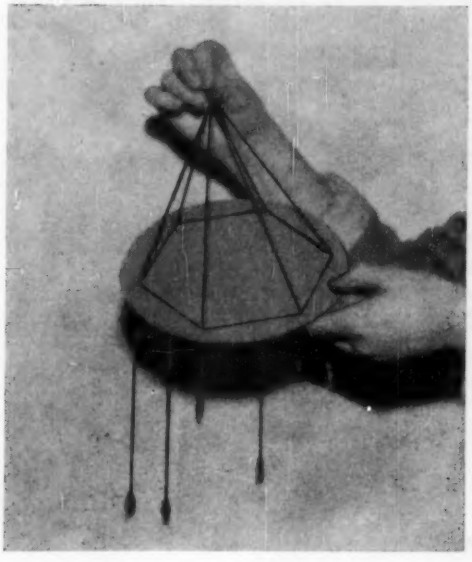
(See Fig. 2.) In the calculus it is shown that the envelope of a family $f(x, y, \alpha)=0$ is obtained by eliminating the "parameter" α from the system of two equations $f(x, y, \alpha)=0$ and $f_\alpha(x, y, \alpha)=0$, where $f_\alpha(x, y, \alpha)$ denotes the partial derivative of f with respect to α. The result here is $(x-y)^2=2x+2y-1$. This is the equation of a parabola since the second degree terms form a perfect square.

Part II. In case the two sticks are not perpendicular, the proof can be reduced to the case just considered by the simple expedient of viewing a perpendicular model sideways at a suitable angle. In technical language, the properties of being a straight line, the envelope of a family of lines, and a parabola are all invariants of this type of projective transformation.

The reasoning used in the preceding paragraph could be applied to the case where the uniform spacing on one stick is different from that on the other. It is also interesting to note that if the sticks are pulled apart, so that they no longer intersect, the lines become a family of rulings of a hyperbolic paraboloid, and the model (that part of it formed by two of the arms, and the rubber bands between them) becomes a model for that quadric surface.

Fishline and sinkers

by Emil J. Berger



A fishline plus a few lead sinkers and a piece of cardboard are all that is needed to produce a dynamic model of a polyhedral angle. Figure 3 indicates how the device may be assembled and manipulated. To produce the device it is only necessary to draw a polygon on a piece of cardboard, punch holes at the vertices, string the fishlines as indicated in the figure, and knot them together to form the vertex. The lead weights will keep all the strings taut. The device is a useful one for illustrating the theorem which states that the sum of the face angles of a polyhedral angle is greater than 0° and less than 360°.

Figure 3

• HISTORICALLY SPEAKING,—

Edited by Phillip S. Jones, University of Michigan, Ann Arbor, Michigan

America's second mathematician: not Adrian, but Adrain

by Harry M. Gehman, University of Buffalo, Buffalo, New York

The interesting article by P. S. Jones on Nathaniel Bowditch published in the May number of this journal[1] mentioned another mathematician of the same period who, like Bowditch, contributed toward the beginnings of mathematical research in this country.[2] Unfortunately the name of the second mathematician was misspelled. His name is not Adr*ian*, but Adr*ain*. I believe that the proper pronunciation of his name is "Add-drain'."

Robert Adrain (1775–1843) is of sufficient importance in the history of mathematics in America to receive at least a line or two in most histories. For example, the Smith-Ginsburg history contains at least eight references to Adrain.[3] A brief biography is given on pages 91–92 and his portrait appears on page 93.

A more extensive account of Adrain's life and mathematical accomplishments was given in 1925 by J. L. Coolidge in his retiring address as president of the Mathematical Association of America. This address was published in the *American Mathematical Monthly* with another portrait of Adrain.[4] Coolidge mentions a proposed biography and critical study of Adrain's unpublished manuscripts to be made by the late M. J. Babb of the University of Pennsylvania, but this was not completed before Professor Babb's death.

Robert Adrain was born in Ireland on September 30, 1775. Given the best available education by his father, he was able to support himself by teaching and acting as a private tutor until the Irish uprising of 1798. Adrain served as an officer in the insurgent forces and was wounded in the back, but managed to escape to the United States with his wife and infant daughter.

After teaching in and acting as principal of academies in Princeton, York, and Reading, Pa., Adrain became professor of mathematics in Rutgers College in 1809. He went to Columbia as professor in 1813 and remained until 1826. After another year at Rutgers, he went to the University of Pennsylvania as professor of mathematics, becoming vice-provost in 1828. He resigned from his positions at Pennsylvania in 1834, returned to New York and taught for four years (1836–1840) in the Columbia College grammar school. He retired to New Brunswick in 1840 and died there on August 10, 1843, sixty-eight years old.

[1] P. S. Jones, "America's First Mathematician," THE MATHEMATICS TEACHER, Vol. XLVIII (May, 1955), pp. 333–338.

[2] R. C. Archibald, *A Semicentennial History of the American Mathematical Society* (New York: 1938), p. 1.

[3] David Eugene Smith and Jekuthiel Ginsburg, *A History of Mathematics in America before 1900, Carus Mathematical Monograph Number 5* (LaSalle, Ill.: Open Court Publishing Co., 1934).

[4] J. L. Coolidge, "Robert Adrain, and the Beginnings of American Mathematics," *American Mathematical Monthly*, Vol. 33 (1926), pp. 61–76.

Robert Adrain taken from D. E. Smith and Jekuthiel Ginsburg, *A History of Mathematics in America before 1900.*

Adrain can hardly be held up to the members of the National Council as an example of a good teacher of mathematics. The following statement is reputed to have been made by one of his former pupils: "If one was thoroughly prepared in his recitation, all was well, but if a student was in doubt or needed a word of explanation in a difficult problem, he not only did not get assistance, but was sent down with some remark of the sort 'If you cannot understand Euclid, dearie (a term he frequently used when out of temper), I cannot explain it to you.' The consequence was that only a small portion of the class could keep up with his course, those who had entered college thoroughly versed in the elements of mathematics and who studied very diligently after they had entered his lecture room, in my class not more than one-fifth of the number. I ought to add, however, that those who went to him in private always found him kind in manner and ready to answer their questions, and help them out of their difficulties."[5] Similarly, Adrain's resignation from his professorship at Pennsylvania was undoubtedly due to his inability to maintain discipline.

On the other hand, great credit must be given to Adrain for his efforts toward the popularization of mathematics in this country. He was active in the promotion of scientific societies, a regular contributor to the mathematical sections of scientific journals, famed as the founder of *The Analyst or Mathematical Companion* (1808), and editor of this and other mathematical journals. He edited the American editions of Hutton's *Course of Mathematics*, adding to a late edition a supplement on descriptive geometry.

Adrain's mathematical publications are chiefly in the fields of diophantine algebra and calculus of variations. His chief claim to fame, however, lies in his having published in the *Analyst* in 1808 the first demonstration of the exponential law of error. In fact, he gave two proofs, the second of which is open to some criticism. But to quote Coolidge: "Nothing can take away from Adrain the credit of having been the first to face squarely the problem of deducing a general law for the distribution of errors, and of carrying it through to the point of finding a formula which, while not perfect, and not always applicable, is still the best that has been devised. Is it too much to say also that this formula was the first broad principle of pure mathematics discovered in America?"[6]

Considering his limited library facilities and lack of contacts with European mathematicians, Adrain accomplished much. To quote Coolidge again: "What he might have accomplished under more favorable circumstances must always remain a subject of conjecture. What he did accomplish entitles him to the glory of a pioneer in the development of American mathematics."[7]

[5] Coolidge, *loc. cit.*, p. 63.
[6] Coolidge, *loc. cit.*, p. 69.
[7] Coolidge, *loc. cit.*, p. 76.

• MATHEMATICAL MISCELLANEA

Edited by Paul C. Clifford, State Teachers College, Montclair, New Jersey, and Adrian Struyk, Clifton High School, Clifton, New Jersey

A readiness program for finding volumes by integration

by Margaret F. Willerding, Harris Teachers College, St. Louis, Missouri

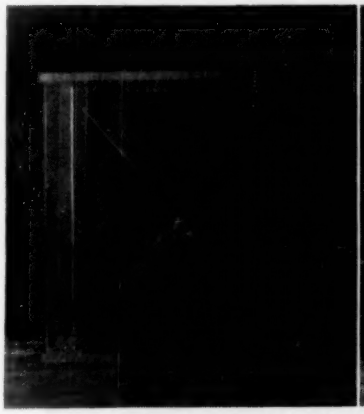
Woody defines educational readiness as the preparation which an instructor consciously makes in getting the student ready to learn the things to be taught.[1] The subject of quadric surfaces in analytic geometry offers a marvelous opportunity for the presentation of a readiness program for the finding of volumes by integration in calculus.

In a readiness program experiences are provided in which the students contact, informally and concretely, processes and ideas that are later to be taught more systematically. The construction of models of quadric surfaces provides excellent experiences for future students of the calculus to become acquainted with solids of which they will later find the volumes by integration.

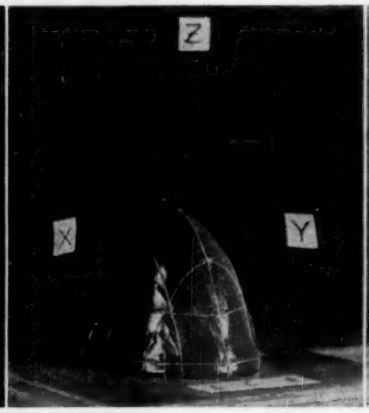
The study of surfaces of revolution is direct preparation for the finding of volumes of surfaces of revolution by integration. In the construction of a model the student can see the cross sections which will later be used in the integration processes. He learns the equations of the surfaces which will confront him later in the calculus.

The general quadric surfaces are concrete examples of "solids with known parallel cross sections" which the student

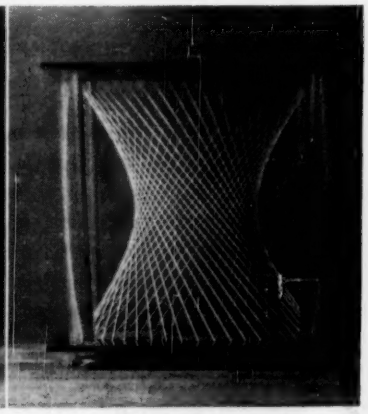
[1] Clifford Woody, "A General Educator Looks at Arithmetic Readiness," THE MATHEMATICS TEACHER, Vol. XXX (November 1937), pp. 314–321.



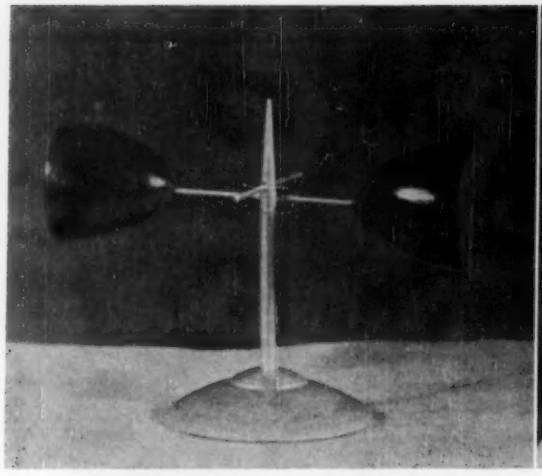
Elliptic cone



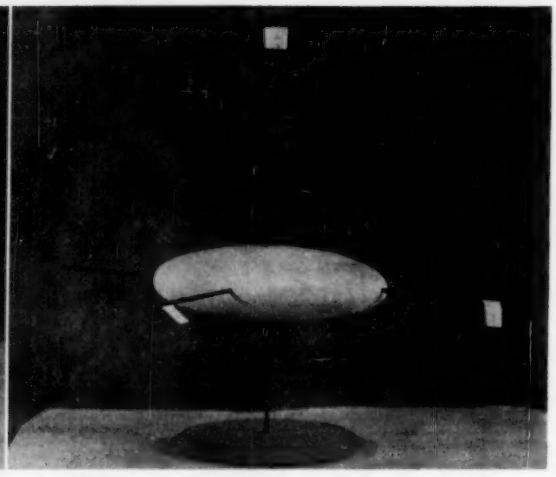
Elliptic paraboloid



Hyperboloid of one sheet



Hyperboloid of two sheets



Ellipsoid

will meet in calculus. The models of these quadric surfaces show the parallel cross sections very clearly.

The photographs show some of the models that an analytic geometry class made. Each student explained his model to the class, giving the equations of the solid itself and equations of the sections parallel to the coordinate planes. In some models which were made of clay, these sections were illustrated by cutting.

The author followed up one particular group to evaluate this simple readiness program. In a calculus class consisting of students who had been exposed to the readiness program and students who were not exposed to the readiness program, it was found that the readiness-program students had far less difficulty with volume problems, particularly those involving a known parallel cross section, than the non-readiness-program students.

The modern trend of combining analytic geometry and calculus in one course offers an excellent opportunity for the presentation of this simple readiness program.

A direct proof of the binomial theorem

by L. M. Weiner, De Paul University, Chicago, Illinois

The standard proof of the binomial theorem is given in most college algebra textbooks by induction. Some of the textbooks use the combinatorial notation in order to shorten the proof somewhat. One who is familiar with the combinatorial notation may, however, avoid a proof by induction and prove the theorem directly by use of the distributive law.

The expression $(x+y)^n$ stands for $(x+y)(x+y)\cdots(x+y)$ written out n times. In evaluating this product according to the distributive law, one takes the sum of the 2^n products, each of which is obtained by choosing an x or a y from each of the n parentheses above. If one chooses i x's, one must choose $n-i$ y's. This gives rise to a term of the form x^iy^{n-i}. Moreover, the number of such terms is precisely the number of ways in which one may choose i x's out of a group of n x's, and this in turn is the meaning of C_i^n. This holds for $i=1, 2, \cdots, n$, and so

$$(x+y)^n = C_n^n x^n y^0 + C_{n-1}^n x^{n-1} y + \cdots + C_i^n x^i y^{n-i} + \cdots + C_0^n x^0 y^n.$$

• MATHEMATICS IN THE JUNIOR HIGH SCHOOL

Edited by Lucien B. Kinney, Stanford University, Stanford, California, and Dan T. Dawson, Stanford University, Stanford, California

What should be the nature and content of junior high school mathematics?

by William David Reeve, Teachers College, Columbia University, New York, New York

Historical review

I do not think that the various departments, so called, in The Mathematics Teacher are equally interesting or equally valuable, but a new department, introduced in the January, 1955, issue, is one that I think should receive the support of all teachers of mathematics whether or not they are actually members of The National Council of Teachers of Mathematics. I refer, of course, to the new department edited by Kinney and Dawson of Stanford University. It will have my full support because I think that we have made a failure, more or less, of the junior high school movement.

The junior high school movement began as such, around 1915, although some individuals interested in mathematics were already experimenting with a new type of mathematics for grades 7, 8, and 9 before 1915.

In its famous 1923 report the National Committee on Mathematical Requirements adopted the following resolution:

> The National Committee approves the junior high school form of organization, and urges its general adoption in the conviction that it will secure greater efficiency in the teaching of mathematics.[1]

The Committee on the Reorganization of Secondary Education, in its pamphlet on the *Cardinal Principles of Education* issued in 1918, had advocated a reorganization of the school system:

> We therefore recommend a reorganization of the school system whereby the first six years shall be devoted to elementary education designed to meet the needs of pupils of approximately 6 to 12 years of age; and the second six years to secondary education designed to meet the needs of approximately 12 to 18 years of age. . . . The six years to be devoted to secondary education may well be divided into two periods, which may be designated as the junior and senior periods.

In accordance with this view, the National Committee on Mathematical Requirements considered together, as a unit, the content of the course of study in mathematics for grades 7, 8, and 9. And, as reported by Kinney in the first article in this new department (see p. 35 of the January issue), the recommended program in mathematics in the junior high school was to consist of "an introductory, basic, exploratory course, in which the simple and significant principles of arithmetic, algebra, and geometry, statistics and numerical trigonometry are taught so as to emphasize their natural and numerous relations."

Content and organization

This is general mathematics in its best and real sense. Moreover, the National Committee recommended that the material described above "should be required of all pupils and that, under favorable

[1] National Committee on Mathematical Requirements, *The Reorganization of Mathematics in Secondary Education* (Boston: Houghton Mifflin, 1923), p. 15.

conditions, this minimum can be completed by the end of the ninth school year."

The National Committee on Mathematical Requirements, in discussing "general" courses, said:

> In recent years there has developed among progressive teachers a very significant movement away from the older rigid division into "subjects," such as arithmetic, algebra, and geometry, each of which shall be "completed" before another is begun, and toward a rational breaking down of the barriers separating these subjects, in the interest of an organization of subject-matter that will offer a psychologically and pedagogically more effective approach to the study of mathematics.
>
> There has thus developed the movement toward what are variously called "composite," "correlated," "unified," or "fused," "general" courses. The advocates of this new method of organization base their claims on the obvious and important interrelations between arithmetic, algebra, and geometry (mainly intuitive), which the student must grasp before he can gain any real insight into mathematical methods and which are inevitably obscured by a strict adherence to the conception of separate "subjects." The movement has gained considerable new impetus by the growth of the junior high school, and there can be little question that the results already achieved by those who are experimenting with the new methods of organization warrant the abandonment of the extreme "watertight-compartment" method of presentation.
>
> The newer method of organization enables the pupil to gain a broad view of the whole field of elementary mathematics early in his high school course. In view of the very large number of pupils who drop out of school at the end of the eighth or the ninth school year or who for other reasons then cease their study of mathematics, this fact offers a weighty advantage over the older type of organization under which the pupils studied algebra alone during the ninth school year, to the complete exclusion of all contact with geometry.[2]

The argument given by The National Committee on Mathematical Requirements for a general course in mathematics in secondary education is as potent today as it was in 1923.

> In the junior high school, comprising grades seven, eight, and nine, *the course for these three years should be planned as a unit with the purpose of giving each pupil the most valuable mathematical training he is capable of receiving in those years, with little reference to courses he may or may not take in succeeding years.*[3]

In the second report of the Commission on Post-War Plans the course for the junior high school is fully discussed.[4] They give the following theses for this period:

Thesis 9. The mathematical program of grades 7 and 8 should be essentially the same for all pupils.

Thesis 10. The mathematics for grades 7 and 8 should be planned as a unified program and should be built around a few broad categories.

Thesis 11. The mathematics program of grades 7 and 8 should be so organized as to enable pupils to achieve mathematical security and power.

Thesis 12. The large high school[5] should provide in grade 9 a double track in mathematics, algebra for some and general mathematics for the rest.

In discussing Thesis 12 the Commission said:

> The situation in the ninth grade of the large high school clearly requires a double track in mathematics—algebra only for those whose ability and future outlook indicate to their advisers that they should take it, and a good course in general mathematics for the rest. General mathematics for the ninth grade is here defined as a course that includes and emphasizes the elements of the functional competence as outlined in the Check List on pages 197–198 of this report.[6]

Thesis 13. In most schools first-year algebra should be evaluated in terms of good practice.

In commenting on Thesis 13 the Commission said:

> Let no one assume that all is well with first-year algebra. There is a wide gap between first-

[2] *Op. cit.*, pp. 12–13.

[3] *Op. cit.*, p. 14. The italics are theirs.

[4] "The Second Report of the Commission on Post-War Plans," THE MATHEMATICS TEACHER, Vol. XXXVIII (May, 1945), pp. 205–208.

[5] By a large high school the Commission means a school of more than 200 pupils. See Ellsworth Tompkins, *Class Size, The Larger High School*, Circular No. 305 (Federal Security Agency, Washington, D. C., 1949).

[6] Commission on Post-War Plans, *op. cit.*, p. 206. See also W. D. Reeve, *Mathematics for the Secondary School* (New York: Henry Holt & Co., 1954), pp. 444–446.

year algebra as it is commonly taught, and what good teachers everywhere have long demonstrated.[7]

For a while after the National Committee Report appeared, some schools made a real effort to experiment with the kind of courses that the Committee had suggested. In this respect the Committee used great judgment for, while they were definite in general procedures, they gave five separate plans (A–E inclusive) for teachers to try out so that the work would not be crystallized too soon. In some cities and towns separate junior high schools were formed, but in many places, even though the name "Junior High School" appeared on the front of the building, one found, if he went inside, the same old traditional arithmetic.

In spite of some progress it must be said that the mathematics of the junior high school in many places in this country is far from satisfactory. Teachers have given lip service to the ideas, but that is about all. Even in places like Minnesota and New York where the state syllabi for the junior high school is excellent, the practice in some schools leaves much to be desired.

Grade placement of materials

While no single order of topics or particular grade placement of materials is absolutely essential, there has come to be a fairly uniform practice in the placing of the larger units within the various years.[8] The following outline suggests what has proved in actual practice to be successful.

Grade 7	*Grade 8*	*Grade 9*
Informal geometry	Algebra	Algebra
Arithmetic	Business arithmetic	Numerical trigonometry
Algebra	Informal geometry	Demonstrative geometry
		Arithmetic

No matter how the material is arranged, if we follow this general scheme we may expect at least two important results. First, the student will have the best possible preparation, within the time allowed, to meet the mathematical needs that confront him in practical life. Secondly, he will be preparing himself in the best possible way to continue his work in mathematics and science if he proceeds further in school.

I would like to second Kinney's suggestion that all those who really wish to see a better type of mathematics in the junior high school contribute articles for this new department.

[7] *Ibid.*, p. 207

[8] *The Place of Mathematics in Modern Education*, The Fifteenth Yearbook of The National Council of Teachers of Mathematics, pp. 246–251. The Joint Commission gives a grade placement chart that wil be helpful in planning a course in general mathematics

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Three do's and one don't

"For effective technical writing, these suggestions from Professor John Foster, Jr., Columbia Graduate School of Journalism:

1. Always remember to whom you are writing. Many good radio and TV writers develop a clear mental picture of some particular person (real or imagined) and write for that one person.
2. Don't assume you are writing to a captive audience that 'ought to be interested.' Everything you write must compete for reader interest.
3. One reason we fail to write interestingly is tradition. Our written language isn't changing as fast as the times. We may talk in the nuclear age, but when we write we suddenly revert to the past century. Today we need direct simple language that lives and gets action.
4. Why not adopt the radio writer's technique to type a line or two, then stop and read it aloud to ourselves? First, it will show us how our writing *sounds*. The second reason the writer reads the stuff over is to make sure he's not getting a sentence so long he'll run the announcer completely out of breath and force him into an awkward gulping pause right in the middle of a thought. Like that one."

—*Taken from* Ed Press Newsletter, *May 10, 1955*

• MEMORABILIA MATHEMATICA

Edited by William L. Schaaf, Department of Education, Brooklyn College, Brooklyn, New York

Maker of universes

A creative genius is rarely appreciated by his contemporaries, nor are his contributions fully understood at the time. Only the mellowing of historical perspective can correct these shortcomings. To be sure, there have been notable exceptions—Leonardo da Vinci, Isaac Newton, Goethe, Beethoven, and Albert Einstein. These, while they still lived, the man in the street somehow knew would profoundly influence mankind for years to come. So we were considerably impressed, a few months ago, by a brief but illuminating evaluation of the impact of Einstein's work as seen by Waldemar Kaempffert, the incomparable interpreter of science. Writing in the *New York Times*, in his well-known column, "Science in Review," Mr. Kaempffert says, in part:[1]

What most cosmologists regard as the finest scientific mind that has appeared since Newton formulated his laws of motion nearly three hundred years ago passed on when Albert Einstein died last week. We may have to wait another three centuries before another mind like it gives science an impetus as powerful as that given by the theory of relativity.

The general public never understood him. It was awed by his intellectual power, but his indifference to the amenities, to conventions, including wealth, his way of forgetting or ignoring engagements led it erroneously to class him with the absent-minded dreamer of fiction. What's this about matter wrapping space around itself, a closed universe, four-dimensional space, it asked? It is still asking, and it is not likely that it will ever understand. Ask the corner policeman to explain why the earth and the planets go around the sun, and he is stumped. But he accepts the fact, which means that he accepts the universe of Copernicus. So it is with Einstein The general public learned to accept his bewildering theories, even though it did not understand them. . . .

A theory is an invention. Like any other invention it must work. Einstein's theories worked long before the atomic bomb was developed. In fact, they worked so well that astronomies had to be rewritten; our conceptions of space and time had to be changed; we had to rid ourselves of the notion that there are forces, an animistic notion that comes down from the time when wise men supposed that the planets were pushed around the sun by invisible angels. . . .

Bertrand Russell once remarked that a mathematician does not have to know what he is talking about. Though the premises may be the wildest assumptions, the logic must be faultless. But the physicist must know what he is talking about. He gives values to the symbols in the mathematician's equations. When this is done and he tests the equations by looking about him at what is happening in the universe he knows whether they are wright or wrong. Einstein's equations were tested in this way, and they proved to be correct.

The universe is not given to us as if it were a table or a brick. It must be interpreted, meaning that we must create it. Everyone creates his own universe. The scientists, especially the mathematicians, do better than most of us because they reason more rigorously and include all known relevant facts about matter, time, motion and space into consideration. So it happens that there are hundreds of possible universes.

Which of these many universes is the "right" one? We shall probably never know. The universe that agrees with observation and experience at the moment is the "right" one for us. When Einstein announced his theory of relativity his universe was in better agreement with what was observed and experienced than was the Newtonian universe which educated men and women accepted for three centuries. . . .

Considered in the light of this sketchy record it is apparent that cosmology and atomic physics of today are largely the creation of Einstein, even though his original conceptions have been modified. No wonder that the head of every scientist was bowed when the news came from Princeton that the most penetrating and fearless intellect of our time was no more.

[1] *New York Times*, April 24, 1955; excerpts quoted by permission.

The abacus and the brain

Scarcely half a dozen years have slipped by since E. C. Berkeley's *Giant Brains, or Machines That Think* quietly ushered in the present era of automation. The contributions of Berkeley, Norbert Wiener, C. E. Shannon, Warren Weaver, and others already belong to the market place. Thanks to skillful interpreters, whose efforts we are in no way belittling, the man on the street is not frightened by such phrases as "information handling," "cybernetics," "binary coding," "feed-back," "programming," "mathematical theory of communication," etc. On the contrary, his imagination has been fired by the notion of "mechanical brains"; he thrills to machines that can "store information"; machines that can "remember"; machines that test logic; machines that play chess and gin rummy; machines that can perform 5,000 additions in a second; machines, in short, which bid fair, so he thinks, to supplant the human brain.

The world of the near future—a world of automation—has been dramatically portrayed by Aldous Huxley and George Orwell, to say nothing of their lesser imitators. But now a sober thinker justly gives us pause. Let us not be naive, he insists. We have a perfect right to ask whether the electronic calculator is conscious.[2]

> Is it capable . . . of imagination? Does it have any curiosity? Can it sympathize with anything? Can it be happy or miserable? Was it ever known to laugh, or even to show, by any unwonted flickerings in its tubes, that it considered something amusing? Does it—and this is most important of all—prefer one thing to another, or does it have its being in a universe where nothing has value, where all things are indifferent?[3]

And he continues relentlessly:

> In any event, to ask the questions either of the machine or of ourselves is to take the first step back in the direction of that crossroads, passed perhaps three centuries ago, when we first began to diverge from the path of Wisdom into the path of Inadequate Knowledge. If we retrace our course, we shall be surprised to discover how much we have tended to forget about ourselves, how little we have studied, or even considered, the most remarkable of our capacities. We may even conclude that the ability to figure or to scheme is so far from being our only unique ability that it is not even the most important one—as indeed the possibility of making a machine which can do it for us sufficiently indicates. Perhaps man is not, first of all, a Reasoning Animal; perhaps something else that he does with his mind is even more obviously unique than reasoning. But what, then, shall we call this other thing; what is it that it is hardest to imagine a machine's doing for us?
>
> We might, I suppose, call it "wanting." Certainly even the stupidest man is capable of desiring something, and the cleverest of machines, no matter how brilliantly it may solve differential equations, is not. But the word "wanting" has a more refined and subtler cousin called "preference," which might do better. Man is an animal who not only wants something tangible but is capable also, even among things as insubstantial as ideas or beliefs, of *preferring* one thing to another. And when one has said that, one has arrived at the conventional terminology of metaphysics: Man has a Sense of Values. Other animals may or may not be capable of something out of which the Sense of Values develops. But a machine certainly is not. And there is the grandest of all the differences. . . .
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> Perhaps, then, those wonderful electronic calculators are not, after all, anything like our brains. Perhaps the best of them is only a superabacus and therefore a triumph of human ingenuity but, no more than the laundryman's convenient device, a real challenge to the human being's uniqueness. To ask which it is—gadget or brain—is at least no academic question.
>
> To answer one way is to take what is perhaps the final step, not merely in the acceptance of mechanism as a philosophical doctrine, but in the direction of a civilization in which men will become more and more machine-like. To answer the other way is to choose instead the working conviction that man, as he was and as he can be, is neither the Economic Man nor the I.Q. Man, but "The Animal Which Can Prefer." It is to believe that the most stupendous of his inventions was not the wheel, or the wedge, or the lever, but the values by which he has lived, and that the ability to act on, for example, the assumption that loyalty is better than treachery even when both seem to give a practical answer to a given problem, is more significant than any other ability he has ever manifested. It is also to believe that, in the future as in the past, what becomes of him will depend less on what machines he invents or what gov-

[2] Joseph W. Krutch, *The Measure of Man* (New York: The Bobbs-Merrill Company, 1954), Chapter 8: "The Abacus and the Brain."

[3] *Ibid.*, pp. 169–172. Quoted by permission.

Continued on page 440

A column of unofficial comment

What makes a good textbook review?

by M. F. Rosskopf, Teachers College, Columbia University, New York, New York

In a recent article Stephen Potter wrote, with tongue in cheek, that the definition of reviewmanship is:

"In its shortest form . . . 'How to Be One Up on the Author Without Actually Tampering With the Text.' In other words, how, as critic, to show that it is really you yourself who should have written the book, if you had the time, and since you hadn't, you are glad that someone has, although it is obvious that it might have been done better."[1]

Some recent book reviews seem to have been written using Potter's succinct definition as a motto: "How to be one up on the author without actually tampering with the text." It is a question whether this is the best motto to have in mind when writing a review.

A second question is: What points should a good review cover? In a discussion written some years ago Dickson noted that reputable mathematics journals in America never accepted a manuscript for publication without having it approved by at least one referee. "Surely book reviews should be submitted to impartial referees prior to publication."[2] So far as this writer knows, Dickson's suggestion has not been adopted. Perhaps it would be well for THE MATHEMATICS TEACHER, in order for the editorial board to secure book reviews that are honest, informative, and fair to the author, to try Dickson's suggestion. That is, extend its referee system to include book reviews.

On the basis of these comments, it is possible to make a few suggestions for book reviewers preparing reviews for THE MATHEMATICS TEACHER.

1. All book reviews are by invitation of the editor.

When a book is received by the editor, he and his staff will choose a person competent in the area covered by the book to write a review. No voluntary reviews will be accepted.

2. All book reviews will be submitted to at least one referee.
3. Does the book show a philosophy or "frame of reference"?

That is, does the book find a place in the present accepted plan of education? Does it deviate from the mainstream by forging ahead or lagging behind? Is it a conservative treatment in its area or is it a departure from usual practice?

4. Does the book show use of a contemporary learning theory in its presentation of concepts?

Does the author make proper use of intuitive methods of development of concepts? Does he make use of generalization-explanation-application procedures? Is the practice material such as to extend concepts further and to develop skills?

5. Is the material presented in the book a mathematically sound development? Are the ideas of contemporary mathematics reflected in the treatment?

[1] Stephen Potter, "The Art of Reviewmanship," *The New York Times Book Review*, May 8, 1955.

[2] L. E. Dickson, "Should Book Reviews Be Censored?" *The American Mathematical Monthly*, Vol. XXX (July-August 1923), p. 253.

A reviewer must be reasonably competent himself in order to judge the competence of others. Hence, it behooves him to consider the textbook presentation carefully before he points out an "error." It seems reasonable that a new book should reflect some progress in mathematics; hence it is believed it is important for a reviewer to search for evidence in the book he is reviewing of modern mathematical ideas and modern treatment of old, familiar ideas. Of particular importance are the definitions given. Are these correct? Does the author describe rather than define when it is not possible to define correctly at the level for which the book is intended? Does he use a sufficient number of examples to make clear to a student the meaning of a word or a concept?

6. What is the content of the book?

A reviewer should do more than merely list the chapter titles. How do the contents differ or deviate from other textbooks of the level under consideration? How are the contents the same?

A few "don'ts" are in order as further suggestions to reviewers:

1. Don't be afraid to be sincerely frank or honest with readers, even to the point of displeasing author(s) and publisher(s).
2. Don't waste space repeating claims of authors or publishers.
3. Don't neglect to polish a review until it is couched in correct language.
4. Above all, don't try "to convince the reader that . . . [you are] a person who knows a great deal about the subject and how it should have been presented."[3]

[3] Dickson, *op. cit.*, p. 253.

What's new?

BOOKLETS

Byroads of Algebra, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D. C. Booklet by Margaret Joseph, 16 pp., 40¢.

How to Study Mathematics, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N. W., Washington 6, D. C. Handbook by Henry Swain, 32 pp., 50¢.

DEVICE

Durex Compass-Protractor-Ruler, Models No. 2 and No. 3, Circline Ruler Company, 4609 Waveland Court, Des Moines 12, Iowa. Combination compass, protractor, and ruler; made of plastic with brass eyelet; 40¢, postpaid; discounts for quantity orders.

FILMS

Plucked Strings (5 min., $10.50), *The Simson Line* (13 min., $28.00), and *The Cardioid* (15 min., $28.00), T. J. Fletcher, Sir John Cass College, Jewry Street, Aldgate, E. C. 3, London, England. Silent mathematical films; B & W; Available in 16 mm. or 35 mm.

FILMSTRIP

Literacy in Mathematics, Bureau of Audio-Visual Instruction, University of Nebraska, Lincoln, Nebraska. 35 mm. filmstrip in color; 33 frames; educational author, Milton W. Beckman; teaching guide included; $5.00.

INSTRUMENTS

Combination Drawing Instrument (No. 7551), W. M. Welch Scientific Company, 1515 Sedgwick Street, Chicago 10, Illinois. Combination ruler-compass-protractor, made of metal and plastic; $3.50.

Pattern Dial (No. 7552), W. M. Welch Scientific Company, 1515 Sedgwick Street, Chicago 10, Illinois. Metal and plastic circular disk for drawing regular polygons or for use as a compass or protractor; $3.50.

MODEL

The Master Cube, Educational Supply and Specialty Company, 2823 East Gage Avenue, Huntington Park, California. Eight hardwood blocks designed by G. Gilbert Scott for demonstrating the cube of a binomial; 4 pp. study guide; $3.95 plus postage.

Reviews and evaluations

Edited by Richard D. Crumley, University of South Carolina, Columbia, South Carolina, and Roderick C. McLennan, Arlington Heights High School, Arlington Heights, Illinois

A statement of editorial policy

Competent people are raising some serious questions about the suitableness of the mathematics being taught in our high schools and in our colleges. Since THE MATHEMATICS TEACHER is the principal instrument whereby these ideas are brought to the teachers of mathematics in the secondary schools, and since its principal function is to work for the improvement of mathematics education, it seems to the editors that it is the duty of THE MATHEMATICS TEACHER to comment on instructional materials for the secondary schools in the light of present mathematical, psychological, and educational trends. Accordingly, in the future the editors will continue the policy of getting competent reviews written by sincere, honest people. Furthermore, the reviewers will be provided the space to write the reviews that, in the opinion of the editors and reviewers, should be written.

The editors feel sure that the readers of THE MATHEMATICS TEACHER will expect reviewers to state their opinions. However, reviewers will be asked to differentiate sharply between opinions and statements of fact. It is the duty of the editors to see that statements given as fact are actually statements of fact. Immediate corrections will be made if a book is misquoted.

On the matter of opinion, however, no "corrections" should be expected. The authors of the books reviewed may, if they wish, use the columns of "Reviews and Evaluations" for a reply to the reviewer. This is common practice among other reputable magazines and it should furnish ample opportunity for authors to defend themselves.

This, in brief, is the editorial policy of THE MATHEMATICS TEACHER as regards reviews of instructional materials. Other comments concerning the review policies with which the editors fully agree can be found in the "Points and Viewpoints" column on page 418 and in the "Letters to the Editor" on page 406 of this issue.

H. Van Engen, Editor
I. H. Brune, Assistant Editor
R. D. Crumley, Departmental Editor
Roderick C. McLennan, Departmental Editor

BOOKS

Algebra, Course 1, Howard F. Fehr, Walter H. Carnahan, and Max Beberman, Boston, D. C. Heath and Co., 1955. Cloth, xi+484 pp., $3.00.

Algebra, Course 2. Howard F. Fehr, Walter H. Carnahan, and Max Beberman. Boston, D. C. Heath and Co., 1955. Cloth, x+502 pp., $3.00.

In reviewing a mathematics textbook, I find subjectivity unavoidable when the pedagogical aspects of the work are being considered. I shall therefore be bold about my subjectivity but would also like to make clear part of the basis for my judgments: (1) A textbook should be as mathematical as possible, consistent with the intellectual maturity of the students. (2) Its purpose is to communicate to students, not teachers, and care should be taken about certain clichés familiar only to teachers. (3) The content and problems, as well as the format, are important for motivation.

In the first sentence of the preface of each volume the authors state that they "would consistently emphasize understanding of the principles on which the processes of algebra are based." They are considerably more successful than most of the current texts, but I feel that they still fall far short of what we should hope for.

There are a number of somewhat unusual and commendable features. Especially noteworthy is the complete avoidance of transposition. Mainly in Course 1, there are numerous sections of "Discovery Exercises" which are de-

signed to lead by a series of questions from familiar facts to some new principle. The concept of discovery is an important one in the psychology of learning mathematics, and it is good to see textbook effort in this direction.

As far as content is concerned, both Courses would seem to contain everything that could be asked for and they include some welcome topics which have not been in many high school courses. Especially to be noted is the early introduction of rectangular coordinates in Chapter 6 (pp. 181–201) of Course 1, with the concepts of slope and y-intercept being developed and used on pp. 194–198; in the next to the last chapter of Course 2 (pp. 443–459) there is a very satisfactory introduction to the differential calculus with applications of derivatives of polynomials to maxima and minima problems and to instantaneous rate.

Practice exercises of a formal sort are very numerous and should be more than adequate for any situation. The matter of word problems is not so satisfactory, although no worse than in other books. I am objecting here to the nature of these problems in that a huge majority of them are clearly puzzles, some disguised and some not. I feel that this is a serious matter, and that a strong attempt should be made to obtain realistic problems which will clearly show the student that algebra has value in practical affairs and that it is not just a bag of tricks. To take just one example from Course 2 (p. 223), consider one of the notorious "rate of current" problems: "Mary can swim 3 miles an hour in still water. She swims ¼ mile downstream, rests 20 minutes, and then returns, the entire trip taking 38 minutes. Find the rate of the current." I cannot imagine how any student, who has not been completely perverted by this sort of thing, can take this problem seriously or view it as illustrating the values of being able to use algebra. Unfortunately, this is not an extreme example but a typical one. Certainly the chief use of algebra is not in solving puzzles but in very useful and important problems, and I think that every effort should be made to illustrate this.

The format and appearance of these volumes are attractive. A great deal of green is used, sometimes for emphasis and sometimes merely for decoration. There is a considerable number of photographs, but I must wonder what these really contribute. Does a full-page photograph of a football game (p. 14 of Course 1) really contribute much to an understanding of the algebraic process of substitution? Does a photograph of a grandfather clock (p. 243 of Course 2) really contribute to an understanding of variation in connection with a pendulum? It might be argued that they certainly do no harm, but I wonder if we may not delude ourselves by this sort of sugar-coating when we should be striving instead to make the meat of the course more palatable and intellectually nourishing. Motivation should not be on such a superficial level.

In regard to mathematical virtues, there are numerous points to be questioned. First of all, I am disturbed by what I consider a violation of the general nature of mathematics. It is a feature of good mathematics to make clear statements of definitions, assumptions, and theorems, and to label these as such. Throughout both volumes, there are many small sections in bold type and marked with green stars. It is clear that these are to be considered important by the student, but they include a hodgepodge of definitions, assumptions, theorems, procedures, and suggestions. I would like to see these various things clearly distinguished. In Course 1, it may not be appropriate to use the word "Theorem," and instead "Rule" or "Law" might be used, but I would want eventually to have the words "Rule" and "Law" clarified within the deductive structure of mathematics. Whether Course 2 is taught before or after a course in plane geometry, students should certainly be mature enough to learn the difference between things which are assumed and those which are proved. Even in Course 1, it seems to me that students should begin to learn about the mathematician's emphasis on precise definition.

In Course 1, there are a number of mathematical details that are not handled to my satisfaction, and I think they could be improved without producing difficulties of some other sort. There is a great deal of trouble about such concepts as literal number, general number, constant, variable, value, etc. beginning on page 5 and continuing throughout. Many teachers will feel that there is no trouble here because these things are treated as they always have been. Here the matter of communication enters. I suspect that the explanations here communicate to teachers only because teachers have been through this material many times and know what is supposed to be communicated. However, for the thoughtful student to whom this is new these explanations would not make any sense. On page 5 is the statement: "Since we are using letters to represent numbers, we call these letters 'literal numbers.'" In a summary following on the same page are the statements: "A general number is a literal number that stands for any one of many values. A literal number that has only one value is a constant. The numbers of arithmetic are constants." The confusion here is between the thing and the symbol, and mathematics should not contribute to such confusion. The fact is that we are dealing with numbers which are necessarily abstract and cannot be written; however, we can write various kinds of names for these abstractions and these are *symbols*. A *symbol* which represents exactly one number should be called a constant, regardless of whether it is a letter or an Arabic numeral. To use the phrase "general number" seems very dangerous because of the connotation that a general number ought to be a number, when it actually is only a symbol in their system. It might be much better to use only the word "variable" in the sense of a letter which may represent any one of a set of numbers. Since the

student will probably, and should, eventually learn this word, why not use it immediately and use it correctly?

On page 11, the directions about order of performing operations are simply incorrect. The rule reads: "When there is nothing to indicate in what order the operations are to be done, first do all multiplications and divisions in order from left to right and then do the additions and subtractions in any order." Actually, if ambiguity is to be avoided, additions and subtractions must also be done from left to right. For example, in the case of $8-1+60$, if we add first, we obtain $8-61$ and then -53; if we subtract first, we obtain $7+60$ and then 67.

On page 54, in a section on checking an answer to an equation, the student is warned "that if you solve an equation and get an answer that does not satisfy it, your answer is not a root." In view of the meanings explicitly given for "solving" (finding the roots) and for "root" (a number which satisfies the equation), this sentence is close to nonsense since it can be translated into "if you find a number which satisfies an equation and which does not satisfy the equation, then it does not satisfy the equation." Clearly the trouble is a shift in meaning of "solve" to "following certain processes which ordinarily lead to roots."

The introduction of negative numbers is handled in a generally sound way, but there are several unfortunate details. The sentence "If a number has a minus sign, it is called a negative number" (p. 94) is incorrect, since $-x$ may be either positive, negative, or zero, depending on the nature of x itself. Subtraction with negative numbers is nicely explained as "the process of finding what number must be added to the subtrahend to get the minuend" (p. 104), but the usual rule is also given and one may very well fear that the rule will take precedence over the more meaningful explanation. Multiplication of two negative numbers is explained only by means of two very weak examples, both of which use the notion of going back in time. It is disappointing that so difficult and important a matter should be handled so weakly, especially when such excellent explanations as the continuity argument are available.

The treatment of systems of linear equations in Chapter 7 is generally done very well, with early relation of graphing to the algebraic treatment. This is marred by one detail: the lack of precision in stating certain definitions. For example: "A system of equations that has only one common solution is called *consistent*" and "We say that a system of equations whose graphs are parallel lines is *inconsistent*" (p. 207). It is clear (to you and me) that these statements refer to a system of *two linear* equations in *two* unknowns, but I can worry about students to whom this is not clear and who are thus encouraged not to be precise and who may have trouble if they try to apply these generalizations to other systems.

The usual difficulties about square roots and the radical sign are commendably avoided, but one small trouble still remains. It is clearly pointed out that the radical sign refers to the principal square root, which is defined as the *positive* square root (p. 276). The word "positive" should read "non-negative," otherwise $\sqrt{0}$ would have no meaning. Zero also creates another trouble on the same page where the impression is created that every number has two square roots. The good discussion (p. 277) of principal square root of x^2 could have been improved by stating that $\sqrt{x^2}=|x|$, since absolute value has already been defined (but little used).

The explanation of division by a fraction is well done (p. 299) and could, if properly handled in the classroom, do much to heal an awfully sore point in many students' experiences. Chapter 12 on square roots includes an exposition of Newton's method (i.e., make any estimate of the square root, divide this estimate into the radicand; the average of the resulting quotient and the original estimate is a better approximation to the root). This method converges rapidly and is easily made meaningful. The more conventional algorithm is included in an appendix, probably as a sop to the traditionalist—I wish it weren't there. Two interesting chapters on historical topics are included. Course 1 concludes with a brief chapter on the trigonometry of the right triangle.

Turning to Course 2, things run a little more smoothly and my criticisms are more about omissions than commissions. By this time, students should be considerably more mature mathematically (especially if they have had a course in demonstrative geometry) and it would seem appropriate at least to expose them to the deductive structure of algebra. In the first chapter, which is a review of the real number system, we find statements of the commutative *law* of addition, etc., and restatements of the *rules* for operations, but still no formal definitions. If it is possible to have tenth-graders learn about geometric definitions, postulates, and theorems, would it be out of place to talk about the commutative *postulate* for addition and to present a *proof* of the *theorem* that the product of two negatives is positive?

In Chapter 3 on linear equations in one unknown, there is an interesting flaw in the sentence, "If an equation has a limited number of roots, it is called a *conditional equation*" (p. 86). I can find no definition for the word "limited," so I can only guess that it means "finite." If so, there is trouble because "$\sin x=0$" is ordinarily considered a conditional equation, yet it has infinitely many roots. If there is some other interpretation of "limited" or if it be argued that this example is not relevant, I would still reply that the sentence is simply not the sort that a careful mathematician should write.

A more general comment is suggested by a statement on page 151: "The product or quotient of two radicals of the same order is the

radical of that order whose radicand is the product or quotient of the radicands." Here again it is my feeling that the book violates the spirit of mathematics by making a difficult verbal statement when the symbolic statement "$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$" would have been vastly superior. In fact, as the authors themselves show, it has been a part of the history of algebra to develop a constantly improved symbolism which makes work easier; now that we have such a symbolism, why not use it to the fullest? This unfortunate tendency is noticed throughout both volumes.

In Chapter 9 on complex numbers, as in many other places, trouble arises in connection with very fundamental things. The number i is introduced by means of rotations. But at the crucial moment the exposition suddenly becomes fuzzy, and it is impossible to discover precisely where i actually came from. At any rate, there is the statement, "Hence we define i as $\sqrt{-1}$" (p. 258). But, as previously mentioned here, $\sqrt{x}$ has been defined as the positive square root of x, and I can find nothing which changes this definition. Therefore the *definiens* is meaningless and this is no definition at all.

The treatment of logarithms in Chapter 12 seems sound, both mathematically and pedagogically. I can only ask two questions of a general nature. First, why not make use of scientific notation for the determination of characteristics? This notation was introduced in Chapter 4 of Course 1 and extended to negative exponents in Chapter 5 of this volume, yet was used but very little. The usual rules for determining characteristic are confusing and troublesome to many students; the use of scientific notation would greatly simplify this problem, as well as showing another value of that notation. Second, why not introduce logarithms earlier, immediately after the full treatment of exponents, and then make frequent use of them in actual computation in connection with other topics? This is related to another trait of this and most other algebras, namely, that most of the problems are concocted so that there is a minimum of computation. This is misleading and pedagogically bad, since most of the real applications of algebra tend to involve less fortunate numbers and much more computation.

There is just one topic whose omission bothers me slightly; that is the topic of inequalities. A good treament of this subject can do much to display the real nature of algebra and to make more meaningful some of the standard topics. It is possible to transform, solve, and graph inequalities and to graph and solve systems of inequalities, and each of these things has both practical and intellectual value. Also, the notion commonly develops that word problems can always be translated into one or more equations. This is frequently not so, as can be illustrated by many of the standard textbook problems which actually result in a system consisting of an equation and an inequality (perhaps as simple as $x>o$).

If a one-sentence summary is in order, I would state the opinion that these volumes are appreciably better, both mathematically and pedagogically, than most currently available, but they are disappointing in that they could easily have been so much better.—*Robert S. Fouch, University of Chicago, Chicago, Illinois.*

The Book of Modern Puzzles, Gerald L. Kaufman, New York, Dover Publications, Inc., 1955. 188 pp., Paper-bound: $1, Cloth-bound: $2.50.

The originality displayed in this interesting book is not surprising if one is acquainted with the background of its talented author. Mr. Kaufman is a practicing architect whose hobbies include those of magician and puzzle-maker.

Although it is not primarily a mathematics book and may supply puzzles for use at parties and fun sessions, as well as for individual pleasure, it is adaptable to mathematics. Mathematics teachers who are interested in maintaining a classroom environment that promotes student interest will find these puzzles helpful in the achievement of this goal if they select proper ones for the teaching sequence. For example, deduction is emphasized in geometry. Is there any reason for not presenting a puzzle that will prompt the student to deduce the proper conclusion? An alert teacher can use such a situation to emphasize the value of following acceptable patterns of deduction.

Spatial relationships of solid geometry require students to group concepts comparable to those needed in solving the knot problems on page 97, and the block problems on page 102.

More than 100 word and logic puzzles should provide the reader, if not his classes, with hours of thoughtful pleasure—*R. C. McLennan, Arlington Heights Township High School, Arlington Heights, Illinois.*

Mathematical Puzzles, Geoffrey Mott-Smith, New York, Dover Publications, Inc., 1955. viii+248 pp., Paper-bound: $1, Cloth-bound: $2.50.

The puzzles within this book, unlike those in the previous review, are primarily concerned with mathematics. They consist of old favorites and many puzzles of the author's making.

If the reader has Mr. Smith's book of the same title published by Blakiston, he will find that it is almost identical to this edition. The page numbers of this re-publication apply to the same topics as are found on corresponding pages of the previous editions. Thus, there is little need to purchase this collection of puzzles if the reader has the Blakiston book. The low cost of the Dover, paper-bound edition might facilitate its purchase by students in a class or math club.

Mathematical Puzzles presents problems in an interesting, conversational manner, with stories for most of the situations. The puzzles increase in difficulty as the reader progresses,

and are selected for use during the study of arithmetic, algebra, and geometry.

A book of this type appears useful in mathematics classes, if the teacher uses it with discretion.—*R. C. McLennan, Arlington Heights Township High School, Arlington Heights, Illinois.*

First Course in Algebra for Colleges, L. J. Adams, New York, Henry Holt and Company, 1955. Cloth, vi+217 pp., $3.00.

As the author states in the preface, this is a text for a beginning course in algebra for college students without previous training in algebra. It covers the topics usually presented in an elementary course in algebra and, in addition, contains a concluding chapter on the number system.

With the exception of the last chapter, most of the text material is well written and the exercises adequate. The author states that he has attempted to stress the meanings of algebraic concepts and processes, to emphasize fundamental principles, and to illustrate the usefulness of algebra as a tool in other subject fields. In general, he has succeeded. The last chapter, however, is too sketchy to be useful. Moreover, the reviewer is of the opinion that college freshmen without previous training in algebra need a more extensive course, but this could be provided by supplementing the Adams text with more advanced material.—*F. C. Smith, College of St. Thomas, St. Paul, Minnesota.*

Functional Mathematics, Book 3, William A. Gager, Lilla C. Lyle, Carl N. Shuster, and Franklin W. Kokomoor, New York, Charles Scribner's Sons, 1955. Cloth, xiii+481 pp., $3.20.

This book is intended for the eleventh grade and is the third in a series of textbooks for high schools. In this series algebra, arithmetic, geometry, and trigonometry are integrated throughout instead of being presented in their traditional separate courses. It is intended that this course would be sufficiently broad to be a good terminal course, and at the same time sufficiently thorough to be good preparation for college mathematics, even for the fields of science and engineering.

Your opinion of the book will probably depend on your objectives in teaching mathematics. If you think of mathematics as a tool subject, important entirely for its use in practical real-life situations, you will find much satisfaction here. Throughout the book the exercises are realistic, demonstrating a great variety of interesting mathematics applications. The topics include approximate numbers; many types of measurement; the lever principle as applied to pulleys, gears and screws; work and power; indirect measurement; interest, taxes, and insurance.

On the other hand if you teach mathematics as a live subject, fascinating for the splendor of its logical development, an outstanding field of human culture worth studying for itself, you may be disappointed. Like the first two books of the series, this book has only an inductive approach to most of the ideas presented. While there is a good place in the learning process for inductive reasoning, how are students going to develop ability in or appreciation of mathematics without thinking through more thoroughly *why* these principles they are using are "true?" The book itself states that "analogy, or reasoning by comparison, is worthless for proof," and yet it repeatedly uses only analogy to present ideas. A number of times it merely states a formula without proof, and then uses it heavily. It seems as though this "Do it this way" method could give only a superficial experience in mathematical thinking.

There is a chapter on "Reasoning with Facts and Figures," which has a wealth of material. If the ideas from this chapter could have been spread out and *used* through the whole book, there would be more hope that the student could benefit, but so many ideas of logic condensed into one chapter, with not enough chance to practice them, would be very difficult for a high school student to digest. For instance, there is a very careful, perhaps laborious, development of indirect reasoning, but the only illustration of it is taken from solid geometry. Incidentally, there are two pages in this chapter on permutations and combinations (with formulas merely stated) which seem to have no connection with the rest of the chapter.

It would be wrong not to mention some of the inaccuracies and inconsistencies found in this book. In introducing zero, negative, and fractional exponents it is not made clear that one is *defining* a formerly undefined symbol, not *proving* anything. In fractional exponents and radicals there is no warning about rules which fail to hold for negative values of the variable. After pointing out well that a circle is a special case of an ellipse, there is later mention of *four* types of conic sections. There is mention of determining the *shape* of a parabola. How do we "determine the shape" of a parabola? In discussing functional relationship between two variables, the fact is not recognized that there may be values of the independent variable for which the function is not defined. Also there are illustrations of "dependence" given which do not lend themselves to an application of "dependence" [function] as defined on page 148; for example, ". . . your health may depend on your food . . . ," page 147. Speaking of ratio as "a comparison of two like quantities by division" rather than just as the "quotient of the quantities" seems confusing. The word "distance" is used in a rule on page 130 and then it is used in quite a different sense in a rule on page 133—an inconsistency that might confuse the student. Although there is a long chapter on measurement, the word "measure" does not seem to be defined.

Let us turn to some of the good points of the book. I have already mentioned the number of meaningful applications of mathematics in the

exercises. It is these that largely give the book its title, *Functional Mathematics*. In the same sense, the photographs are good and make a meaningful contribution. Sometimes the student is led very well by a series of questions to develop an idea himself. The careful and consistent use of the principles of approximate numbers is excellent. I like the use of scientific notation in introducing logarithms; the directions for the student to make his own slide rule; the use of "triangle of reference" in using the general definition of the trigonometric functions; interest compounded continuously leading to e, natural logarithms, and the growth formula. A protractor is provided for measuring angles in radians and mils. Many teachers teach the theory of these units without ever actually measuring angles in mils or radians. The use of this protractor could well give more meaning to these units.

There are three chapters about compound interest, annuities, appreciation and depreciation of values, taxation, and insurance. Opinions differ as to whether this material, which usually contains a negligible amount of mathematics, belongs in the mathematics course.

As one asks which type of student could benefit from using this book, it seems as though it must be limited to the mediocre middle group. The poor students would be unable to read the lengthy discussions, and the good students would not be sufficiently challenged by the frequently superficial treatment of the "why's."—*Henry Swain, New Trier Township High School, Winnetka, Illinois.*

One Hundred Mathematical Curiosities, William R. Ransom, Portland, Me., J. Weston Walch Publisher, 1955. Paper, viii+212 pp., $3.00.

This book is a compilation of problems of the type that teachers like to give students as "interest arousers" and that students enjoy using to "stump" the teachers. The problems are not original with the author; many of them are of the well-known trick variety such as ones about the bookworm, the frog in the well, and freak cancellations. But the feature that makes the book of most value is the inclusion of those problems that usually are not found in a book of this type: the Peaucellier linkage for drawing a straight line, theorems of Menelaus and of Ceva, constructions (such as those of Mascheroni, Halstead, and Gauss) with limitations on the instruments used, and many others of equal merit.

The commendable features of the book are: (1) the inclusion with each problem of a discussion and at least one solution, (2) a range in difficulty from junior-high into college-level mathematics, (3) an occasional brief historical background of a problem, (4) a profusion of diagrams to help clarify problems, and (5) an alphabetical and a topical index.

The book is workbook size and the paper is not of a very high quality. In spite of a few misprints and repetition of a story (the story on page 77 is repeated on pages 114–115), teachers will find this book quite useful in adding an extra "sparkle" to their classwork.—*Julia Adkins, Ohio State University, Columbus, Ohio.*

Plane Geometry, Arthur F. Leary and Carl N. Shuster, New York, Charles Scribner's Sons, 1955. Cloth, ix+510 pp., $3.80.

A very wise superintendent of schools always said, when after months of study our department recommended a new mathematics textbook, "I will order enough copies for one class, and then have someone in the department try it out for a year or two before you definitely decide that this is the book you want." I have never forgotten the lesson I learned. I cannot be sure of the worth of a book to me until I use it in a class situation. That I have not taught a class using this text you must consider as you read.

The authors are qualified as to experience to write a textbook in geometry. Mr. Leary is a teacher of secondary school mathematics; Mr. Shuster has taught in secondary schools and is now in a teachers' college. The backgrounds of both men undoubtedly account for the wealth of material on applications, their rigor, and technical nature. Whether or not this abundance appeals to you depends on the locality in which you teach and the objectives pertinent to your particular geometry classes. By and large, these applications would not interest girls.

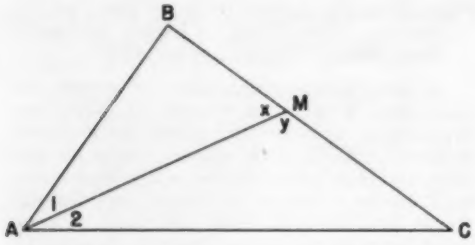
Modern trends are away from the geometry patterned on Euclid's Elements. This book is up to date in this respect. The language is simple, clear, informal, and suited to the age of the student who will study it. It is a decided improvement over the old type book which consists of theorems and problems with few or no explanations. In it geometry grows from definitions and assumptions and becomes a logical whole. Many students will find it self-teaching.

This book is larger than many geometry texts. The pages are large, but the print is, too, and the margins; therefore there is no more material on a page than in the average text. It is well bound and the paper is good. The number of illustrations is adequate, and they are well chosen.

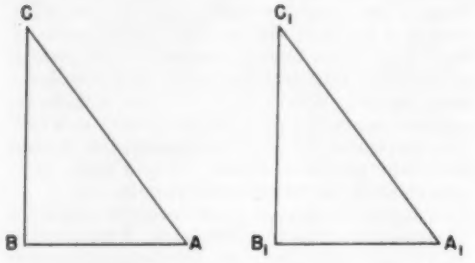
One innovation that I like very much is the use of color—red—to focus attention on the important parts of drawings, on new terms which are defined as need for them arises, on assumptions and important theorems. This aids greatly in looking, say, for a definition, as one thumbs back through the book. However, this pattern should be consistent. For example, new terms are defined on page 54, and none are in red. Similar omissions could be pointed out in many places in the book.

On page 10, different ways for naming angles are discussed, among which are $\angle x$, $\angle y$, and $\angle A_1$, $\angle A_2$. I do not object to the latter but see little reason for it (I would say $\angle 1$ and $\angle 2$)

unless you have a complicated figure and need to talk about many angles. The only figure of this sort that I find is on page 162, Example 6. On page 136, the author quite adequately uses small letters inside the angles.



Then, too, in this same connection, on page 67 we have two triangles lettered *A B C* and A_1 B_1 C_1. I fear here that the student may become confused and wonder why the subscripts are not inside the angles.



On pages 24 and 27, I object to the capital letters *A*, *B*, and *C* being used inside the angles. On page 196 (bottom), we find that "DE means chord DE," but such designation is seldom used. Theorem 44, page 181, should have its figure and proof on the same side of the page. If, in theorem 43, page 180, we are rigorous enough to prove that lines *j* and *k* are not parallel and will therefore intersect, then in theorem 44, we should also prove that *q* and *s* are not parallel and will therefore intersect.

I am not sure that I would give space in a textbook to "Pitfalls to be Avoided," as appear on pages 88 through 93. It is rather early in the game to expect the student to be discerning. Pupils often fix in their minds the incorrect procedures if we point them out before they are made.

These criticisms are minor and may be a matter of opinion. The decision is yours when you examine the book.

I like to see summaries stand out in a book, because students must constantly refer to them. By and large, the summaries here, such as those on pages 152, 177, 119, 184, and 207, are tucked in inconspicuously among other material. I like them to begin at the tops of pages, or be separated by more space from the rest of the printed matter, or printed in heavier type. I vote for well-constructed usable summaries at the ends of the chapters. Chapters 7 and 9 have no summaries, nor is there one at the end of Chapter 6.

Modern trends in the teaching of mathematics impel a good teacher to make constant use of the inductive method in discovering what these authors call "hypotheses" (pages 35 and 82) which are subsequently postulated or proved. Chapter I makes excellent use of induction; and throughout the book countless facts are discovered inductively in exercises which are later proved or assumed. The word "induction" appears early in the book on page 44 and again on page 56, Example 9. It is defined again on page 82, and a very good discussion can be found on page 490, rather near the end of the book. Perhaps the authors are leaving to the teacher the use of the word as they employ the process throughout the course.

In this same connection, the congruence theorems are postulated, and then examples given to show their validity (pp. 44, 45, 46, 47, 48, for example). Then on page 64 we find the method for constructing a triangle, "given side-angle-side." I think the authors have missed a splendid opportunity here for letting the students discover their own congruence theorems by induction, by introducing the constructions first, and by experiment, deciding which combination of parts determine the size and shape of a triangle. True, this process is hinted at on page 35 in Examples 9 and 10, but it seems to me that, from time to time, the induction should just precede the actual proof or assumption to point it as a means for discovering a hypothesis.

As another example, the theorems in Chapter 7, "Angles Measured by Arcs," are ideally discovered by experiment, not only how the angles are measured, but the different kinds of angles which can be formed.

In Chapter 7 also, we find this statement on page 240, third paragraph, "As vertex *X* continues outside the circle, arc *CD* becomes a negative quantity, having decreased to zero and continued below zero." More explanation is needed here, although the authors may feel that this is a function of the teacher. Also, I should prefer that the generalization involved here be placed after theorems 63, 64, and 65.

On page 295, analysis and synthesis are defined, and on page 296 an example of analytic reasoning is given. Since analysis is one of the most important tools we can give a student to encourage the type of reasoning that will help him discover a method for proof, it should be developed early in the course. In many instances, as in theorem 2 on page 98 and theorem 17 on page 140, plans of attack are suggested, but this is not analysis as opposed to synthesis. Theorem 1 on page 94 offers a beautiful opportunity to introduce this way of "discovering" a plan of attack.

The authors of this book may well say, at this point, "This is not a book on how to teach geometry. It is a textbook, and we leave it to

the teacher to lead the student to develop hypotheses inductively and to teach them the analytic approach." The point may be well taken. Perhaps I am looking for the too ideal text for the beginning teacher.

The synthetic proofs of the theorems presented here are, to my way of thinking, informally done. See any proved theorem. I think that this is fine as the student starts his study of geometry. But as he proceeds, I should like his proofs to become more rigorous. I like steps and reasons numbered, and the reason and the step which belong together numbered the same. Students tend to be sloppy, not only in their thinking but in the manner in which they express their thoughts. Insistence on this manner of writing proofs aids in teaching them clarity and preciseness of thinking.

The references to the history of geometry are good and well placed. Sufficient references and analogies to space geometry are made. The chapter on arithmetic and algebra is valuable in aiding the pupil at a time when such review is timely and to show him the interrelation between these branches and the geometry which he is currently studying. It introduces trigonometry after proportions, showing a good practical application. It includes enough analytic geometry to challenge the student who is ready for it. I do not like the chapter on loci placed so near the end of the book. I should prefer to see locus introduced with theorems 2 and 3 on pages 98 and 100 and continued wherever occasion arises.

This book contains a wealth of material, probably more than any one teacher can use in a year's course in plane geometry, but it should meet the needs of all. I believe that any experienced teacher, who teaches geometry meaningfully, will be interested in it as a textbook. Should there be a manual of instruction with recommendations as to a minimum course for the beginning teacher?—*Gertrude V. Pratt, Central Michigan College, Mount Pleasant, Michigan.*

Statistical Methods for the Behavioral Sciences, Allen L. Edwards, New York, Rinehart and Company, Inc., 1954. Cloth, xvii+542 pp., $6.50.

That students in the behavioral sciences rarely possess mathematical training beyond college algebra is widely experienced. The author of *Statistical Methods for the Behavioral Sciences* meets this problem by including brief presentations of needed topics from elementary algebra and such more advanced topics as logarithms, probability, analytical forms of the straight line, and the use of logarithmic and semi-logarithmic paper. His method of presentation throughout the book is designed for the student of moderate mathematical knowledge and maturity. Formulas are developed empirically or algebraically or in both ways, and basic concepts are thoroughly illustrated with numerical examples.

Topical coverage includes the common measures of central tendency and variability, normal distribution, the product-moment correlation coefficient and other measures of association, elementary sampling theory, analysis of variance, and the χ^2 (chi square) test of significance.

Each chapter contains a sufficient number of exercises to provide practice in the use of principles presented. Some of the exercises consist of abstract developments and proofs. A bibliography of four and one-half pages and a summary list of all formulas are found at the end of the book. Fifteen tables, and answers to all exercises, are provided in the appendix.

Readers familiar with Professor Edwards' *Statistical Analysis for Students in Psychology and Education*[1] will have a strong inclination to compare *Statistical Methods* with it. Indeed, there are many points of similarity between the two books with respect to both content and style. Most of the topics in *Statistical Analysis* appear in *Statistical Methods*, but with amplification and extension. In a few cases the orders of topics are different. The chapter on linear regression, which precedes introduction of the product-moment correlation coefficient, and the chapter on random errors of measurement may, for all practical purposes, be considered as topics not found in the earlier book.

Excellent coverage of appropriate material and the manner of its presentation make *Statistical Methods for the Behavioral Sciences* a textbook of promise for use in the area for which it is designed, and one of value in other fields as well.—*Clarence H. Heinke, Capital University, Columbus, Ohio.*

The Theory of Numbers, Burton W. Jones, New York, Rinehart and Company, Inc., 1955. Cloth, xi+143 pp., $3.75.

One of the most striking things about this book is the profusion of excellent problems, many of which are quite original and contribute greatly to its usefulness as a textbook. With a few exceptions, the subject matter is conventional, ranging over congruences, diophantine equations, continued fractions, and quadratic reciprocity. Rather surprisingly, the first chapter is devoted mainly to a development of the number system from the integers to the complexes. The book is well written and will be found useful. Nevertheless, this reviewer submits that the time is now propitious for someone to write a book on elementary number theory introducing modern algebraic concepts, which arise here perfectly naturally, in order to achieve a more fundamental understanding of the subject and to reveal in it some sense of direction.—*W. E. Jenner, Northwestern University, Evanston, Illinois.*

[1] Allen L. Edwards, *Statistical Analysis for Students in Psychology and Education* (New York: Rinehart and Company, Inc., 1946).

• TIPS FOR BEGINNERS

Edited by Francis G. Lankford, Jr., Longwood College, Farmville, Virginia

Helping pupils use proofs of theorems in geometry

by Francis G. Lankford, Jr., Longwood College, Farmville, Virginia

There are at least two important reasons for teaching geometry. One is to help pupils acquire an understanding of the basic properties and relationships of geometric figures and to develop an ability to use them. A second reason for teaching geometry is to help pupils understand the nature of deductive proof and to develop an ability to use this method of proof in establishing conclusions to geometric theorems. The first of these objectives is dominant in the intuitive geometry which is taught as a part of general mathematics in the junior high school grades. The second is the dominant objective in the courses in demonstrative geometry taught in senior high school. In earlier years pupils pursued this second objective through a study of the proofs of many theorems displayed in their textbooks. They also studied corollaries and exercises for which they had to develop proofs more on their own. In more recent years some textbooks in geometry have left the proofs of more of the theorems to be developed by the pupils. Complete proofs are given for enough theorems to serve as models and to help pupils learn how to handle several types of proof such as direct and indirect proofs. This development is in the right direction, for it tends to prevent students from depending on memorization of textbook proofs to get through geometry. Certainly we have all heard the claim too often, "I could never have passed geometry if I had not been able to memorize the proofs." The remainder of this article contains a few further sugggestions for avoiding this superficial outcome.

The first suggestion is that you help pupils learn how to study model proofs found in their books. By model proofs we mean the usual statement of a theorem, followed by a figure containing parts added for the proof, which is then followed with a list of what is given, what is to be proved, and the proof with steps and reasons given. Study of such a model proof should include: (1) a careful reading of the theorem until all terms are clearly understood and until the pupil can separate the theorem into its hypothesis and conclusion. He may need to examine the drawing and then compare his analysis with the "given" and "to prove" found in the model proof. (2) The pupil examines the proof to see what *method* is used to establish the conclusion. For example, if lines are to be proved equal, are they to be proved corresponding parts of congruent triangles, sides of an isosceles triangle, opposite sides of a parallelogram, or by some other means? (3) Examine carefully the steps of the proof and supporting reasons to be sure they are understood—not merely to remember them. (4) It is now helpful in making a model proof a part of one's own thinking to consider some other sequence in the steps of the proof that would still be logically sound. A pupil may even consider varying the model proof by starting with somewhat different helping

constructions. For example, if the model proof for the theorem, "If two sides of a triangle are equal, the angles opposite these sides are equal," starts with the construction of the bisector of the vertex angle and proceeds to prove triangles congruent by showing that two sides and the included angle of one are equal to two sides and the included angle of the other, a student may consider an optional proof by starting with a figure like the following in which ABC is the given triangle and BD is drawn equal to CE. (5) Finally the student completes his study of the proof by fixing in mind only two things: the method of proof and the helping lines that must be added to the figure that represents the hypothesis.

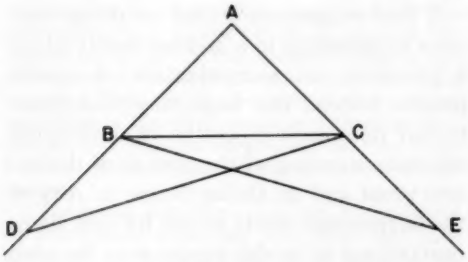


Figure 1

In these suggestions it is assumed that a pupil can read any theorem and decide what is its hypothesis and conclusion. This ability should be developed through careful teaching early in the course in geometry. This may be done by using in class groups of eight or ten theorems. These may be duplicated or placed on the blackboard. Then teacher and pupils together write the hypothesis and conclusions of each theorem. Those theorems which appear later in the course are good for this purpose, as are corollaries. Examples of various ways of stating theorems should be used, such as this conditional statement: if one acute angle of a right triangle is 30°, the side opposite this angle is one-half the hypotenuse; or this declarative sentence: an exterior angle of a triangle is greater than either non-adjacent interior angle. Following such class discussion, pupils may then be asked to write the hypothesis and conclusion of other unfamiliar theorems on their own.

Likewise pupils should early be helped to develop the ability to examine any theorem and draw the figure which represents the hypothesis. The same groups of theorems which have been used once to help pupils learn how to write the hypothesis and conclusion may now be used to help them draw a figure to represent the hypothesis. Class discussion followed by individual pupil exercises may also be used for this purpose.

Now with pupils having developed the ability to write the hypothesis and conclusion and to draw the figure representing the hypothesis, it is unnecessary for them to try to remember these parts of a model proof which they study in a book. These parts of a proof can be done as a pupil puts a proof on the blackboard or writes it in a test. It is with this assumption that it was suggested earlier that pupils need to remember from their study of a model proof only the helping parts that must be added to the basic figure, and the method of proof employed.

Now comes the demonstration of the model proof studied. Frequently, this is arranged by having a pupil go to the blackboard and "put up a theorem" which is later "explained" to the class by the pupil. Such a procedure has doubtful value. For the pupil who listens to the "explanation" (often merely read from the proof written on the blackboard) by a classmate, he gains nothing if he already understands the proof. If he does not, the routine explanation that he hears is too rapid for him to get anything more than another practice exercise in memorizing the proof. Such a demonstration may be varied in several respects to make it more effective. One variation which has proved helpful is for the teacher to write a theorem on the blackboard. Then he may have every pupil at his seat write the hypothesis and

conclusion and draw the figure that represents the hypothesis. Next a single pupil may go to the board and orally explain the method to be used in the proof. He does not write any steps in the proof. Finally all pupils complete the proof at their seats. Now the teacher may suggest an optional proof by using a different helping line in the original construction. Consider the theorem: the sum of the interior angles of a triangle is a straight angle. The model theorem the pupils studied in their textbook may have used a figure such as Figure 2. The teacher may suggest that the

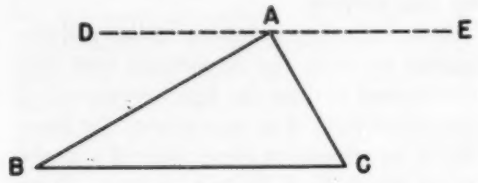


Figure 2

theorem be proved with a figure like Figure 3. The pupil who has merely memorized

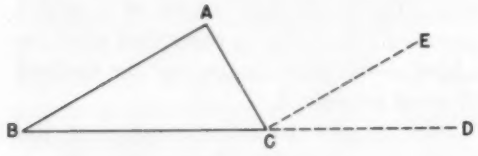


Figure 3

the model proof will likely be confused by this change. The pupil who understands the *method* of proof used in the model proof will have little difficulty in applying it to the slightly different figure.

Another variation that may be used in the demonstration of a model proof studied is to have pupils suggest different sequences in the steps of the proof. This may be done sometimes through oral discussion and at other times with individual seat work. Soon pupils will learn that they may vary the sequence of steps in a proof somewhat but not at random. Such an activity stimulates thoughtful analysis of a proof as opposed to mere memorization of steps.

In following this suggestion that varying model proofs may stimulate independent pupil thinking, one caution must be observed. Pupils must be taught to avoid fallacious conclusions reached through reasoning in a circle. That is, a pupil may propose proof Y for theorem A whereas the textbook model used proof X. Upon careful examination it may develop that in the pupil's proof Y, a theorem was used which required theorem A in its proof. This is reasoning in a circle which is faulty reasoning. However, when a pupil considers an alternate proof and later abandons it because it includes reasoning in a circle, he is clearly enjoying a valuable experience in independent thinking.

A final suggestion is that we design our tests in geometry in a manner not to place a premium on memorization of model proofs. Instead our tests may contribute to our dominant objective of developing an understanding of the nature of deductive proof and an ability to use it. Any of the suggestions made above for oral demonstrations of model proofs may be used in a test. Another variation may include on the test the necessary steps for a proof with a requirement that the pupil arrange them in a logical order. Then, of course, tests may include some so-called "originals." These are usually taken from exercises or corollaries for which model proofs are not given in the pupils' books. Finally, a good but rigorous review test is to require pupils to prove every theorem used in support of the proof of a theorem given in the review test. Such an exercise usually requires more time than the usual class period.

Assuming that beginning teachers will be interested in examining alternate proofs of a few standard theorems, the next issue of "Tips for Beginners" will contain several.

Words! Words! Words!

"I like the words 'epitome,'
'Digamma,' and 'baleen.'
I toss them off with nonchalance,
But don't know what they mean."
—*Sara Garrett*
from the New York Sun

"Absolute value" is a term which appears relatively late in the development of the elements of algebra. Early books, such as Bonnycastle's *Algebra*,[1] do not recognize the need for the term. The rule for adding two numbers of unlike sign, such as -9 and $+4$, was simply stated in terms of "take the sign of the larger number." After the idea of ordering positive and negative numbers came into its own, the rules based on "taking the sign of the larger number" became thoroughly inadequate. (According to the old rule, -9 is larger than $+4$. But, if position on a number line is considered, it is obvious that -9 must be less than $+4$.)

Today's elementary algebra texts tend to define the absolute value of a number in terms of "the number regardless of the sign." This terminology is also thoroughly unsatisfactory. The little mark "$-$" placed before the symbol "6" is as much a part of the number -6 as is any other part of the number—the tail of the 6, for example. The definitions based on "the number regardless of the sign" make as little sense as a definition of absolute value based on "the number regardless of the northeast corner of the number."

The usual definitions can also be criticized on pedagogical principles. It is important that the pupil recognize the difference between the numbers of arithmetic, with which he is familiar, and the numbers of algebra, that is, the positive and negative numbers. The "regardless of sign" definition does not highlight this distinction. Neither does it follow the sound pedagogical principle of basing definitions on the mathematical concepts already in the pupil's possession. Herein also lies the weakness of the definition found in some college texts. Some of these texts define the value of x as:

$$\begin{aligned} |x| &= x; \quad x > 0 \\ &= 0; \quad x = 0 \\ &= -x; \quad x < 0 \end{aligned}$$

While formally correct, such definitions cannot be used for concept developing purposes in the ninth grade.

A more fruitful approach to a definition of the absolute value of a number would seem to be based on developing the distinction between the numbers the pupil has used in arithmetic and positive and negative numbers. (While the unsigned numbers are isomorphic to the positive numbers, such concepts do not seem to contribute to establishing this fundamental concept in the secondary schools.) Once this distinction has been grasped, it is then possible to define absolute value of a positive or negative number as follows: In order to find the absolute value of -10, write down the triplet $(+10, 10, -10)$. Then the unsigned number 10 is the absolute value of both the -10 and the $+10$.

An alternative definition would be the following: To every positive and negative number there is associated an unsigned number which tells how many units the given number is located to the right or the left of the origin on the number line. This unsigned number associated with every positive and negative number (and zero), is the absolute value of the positive or negative number (zero) under consideration.—*Henry Van Engen.*

[1] Bonnycastle, John, *A Treatise on Algebra; In Practice and Theory* (London: Printed for J. Johnson and Company, 1813).

The Washington meeting, December 27-30, 1955

"Santa Claus is coming to town," but so are a lot of mathematics teachers for the December meeting of The National Council. And they'll be bringing their families too, because at the Sheraton Park Hotel where the convention will be held, room rent is free for children under fourteen if accompanied by their parents. They'll be coming early too, because there will be sightseeing trips to Mount Vernon, Arlington, Smithsonian Institution, the Library of Congress. There will be trips to the Bureau of Standards where they will see the standard units of measure for this country and also the electronic computer in action. A special mathematics tour of the White House has been arranged. There's an Embassy Tour too, and you can have trips to several foreign countries because the Embassy soil is considered the soil of the country owning it! The Pageant of Lights, a one-week Christmas Festival last year in Washington, was considered so outstanding that it will be repeated this year and you should not miss it. (Do bring color film.)

Of course, there will be a splendid program. You can't afford to miss the convention and seeing old friends and making new ones. Programs will be mailed to you in November, so do get your registrations in early!

The mathematics teachers of the Greater Washington area, your hosts and hostesses, will be at the gate looking down the road for you!—*Veryl Schult, Local Chairman*

• WHAT IS GOING ON IN YOUR SCHOOL?

Edited by John A. Brown, University of Wisconsin, Madison 6, Wisconsin, and Houston T. Karnes, Louisiana State University, Baton Rouge 3, Louisiana

Mathematics and guidance in Kansas schools at the ninth-grade level

by Earl S. Elliott, 1225 Winfield, Topeka, Kansas

Many school administrators recognize the importance and value of guidance services to the individual. However, relatively few pupils to date have had the opportunity to reap the values of a guidance program.

The most significant and important recommendations for the development of a more functional mathematics curriculum recently have been the reports of the Commission on Post-War Plans in Mathematics. The first report of the commission recommended that:

1. The school should insure mathematical literacy to all who can possibly achieve it. . . .
2. We should differentiate on the basis of needs, without stigmatizing any groups and should provide new and better courses for a high fraction of the school's population whose mathematical needs are not well met in the traditional, sequential courses. . . .
3. We need a completely new approach to the problem of the so-called slow learning pupil. . . .
4. The teaching of arithmetic can and should be improved. . . .
5. Sequential courses should be greatly improved. . . .[1]

The second report of the commission, reported by Raleigh Schorling, emphasized that the school must provide sound mathematical training for our future leaders of science, mathematics, and other fields; and also insure mathematical competence for the ordinary affairs of life.[2]

The fourth and final report of the commission was published as a guidance pamphlet, entitled *Guidance Pamphlet in Mathematics for High School Students*, to make clear what mathematics has to offer to the pupil. Directed to the pupil, it pointed out the importance of mathematics in certain jobs. In addition, it included a list of 29 competencies, a check list of specific mathematical attainments, which constituted the objectives for the mathematical instruction of all pupils.[3]

What are we doing in Kansas in terms of mathematics and guidance at the ninth-grade level? The writer found in a survey of 195 first-class city schools, second-class city schools, rural high schools, county-community high schools, and common school districts that in general we lack an adequate guidance program in ninth-grade mathematics.

Ninety per cent of the schools in Kansas that reported in this study required every ninth-grade pupil to take a course in mathematics, and 98 per cent of the pupils enrolled in the ninth grade were enrolled

[1] Commission on Post-War Plans, "The First Report of the Commission on Post-War Plans," THE MATHEMATICS TEACHER (May 1944), 37: 227–231.

[2] Raleigh Schorling, "The Second Report of the Commission on Post-War Plans," THE MATHEMATICS TEACHER (May 1945), 38: 195.

[3] Commission on Post-War Plans, "Guidance Report of the Commission on Post-War Plans," THE MATHEMATICS TEACHER (November 1947), 40: 315–340.

in a course in mathematics. Courses in general mathematics and algebra enrolled 97 per cent of the total number of pupils enrolled in mathematics.

Sixty-seven per cent of the schools offered at least two courses in mathematics at the ninth-grade level. Courses included were algebra, general mathematics, and business arithmetic. However, 65 per cent of the total number of schools offered only algebra and general mathematics. While almost all ninth-grade pupils take a course in mathematics, only about two-thirds of the pupils enrolled in these schools have a choice of mathematics courses to take in the ninth grade. Two-thirds of our Kansas schools are providing a double-track program in mathematics at the ninth-grade level. Very few schools are providing more than a double-track program.

Thirty-six per cent of the Kansas schools indicated that they employed a guidance counselor in their school. Of the schools with guidance counselors, the median number of periods per day that the guidance counselor had for guidance activities was two periods. The guidance counselor either did not exist in most Kansas schools or had limited time assigned for guidance activities.

Only about one-fifth of the schools made use of tests in placement of pupils in mathematics at the ninth-grade level. Of 50 tests that were used, there was a total of 18 different kinds; therefore, the frequency of any particular test had little significance. It did indicate that few tests were used in Kansas schools for placement, and that the variety of tests used was wide.

Less than one-third of the schools reported having a remedial mathematics program. In some schools general mathematics functioned as general education, in others, as a course for deficient pupils, and in still others as a course for those not going to college. A few schools indicated that remedial work was included as a part of the regular classroom work. One or two schools used a remedial program of a temporary nature in which pupils carried regular or light loads and had extra help until any deficiency was made up.

What can be done to provide adequate guidance for pupils in the ninth-grade mathematics program? First, and probably most important to note, this is not one of the primary functions of a guidance counselor. A guidance counselor can help, but it is a job for the pupil and his parents, the mathematics teacher, and the school administrator.

It is important to recognize that many guidance services and activities are available in schools today that may not be considered a part of a guidance program. The most important reason for organizing these activities would be to make them available to all pupils at a time when they need them most.[4]

If one of the important functions of the school is to provide education with guidance for all pupils at the time they need it, then:

1. Schools should continue to develop the double-track program in mathematics at the ninth-grade level.

2. General mathematics should be developed as an exploratory course in the field of mathematics as a part of general education.

3. Algebra should be developed as preparation for higher mathematics.

A program of remedial instruction should be included on an individual or small group basis for pupils deficient in mathematics. In addition, it should be recognized that all pupils cannot attain the same level of mathematical competence without overteaching or underteaching the extreme groups of pupils, so that a minimum level of competence in mathematics must be flexible and commensurate with the ability of the pupil.

4. Assignment in the ninth-grade mathematics program should be made on the basis of (1) prognosis tests in algebra, (2)

[4] Clifford P. Froelich, *Guidance Services in Smaller Schools*, p. 11.

scholastic ability, (3) previous marks in mathematics, and (4) pupil's choice.

5. Schools should develop the concept of curriculum assignment committees composed of the administrator, mathematics teacher, guidance counselor, and the pupil to provide composite judgement and reviewing relative to the assignment of the pupil to the ninth-grade mathematics program during the preceding year.

6. Schools should develop the use of a personal interview during the preceding year to aid in determining the assignment of the pupil in the ninth-grade mathematics program.

7. The guidance counselor should function as an aid to the curriculum assignment committee.

8. Schools should provide adequate guidance services, and provide the guidance counselor with sufficient time to perform and coordinate guidance activities.

Challenge to the gifted[1]

by Norman Clark, Kirkland, Washington

When Dick Fiddler, a senior at Lake Washington High School, was named one of the 40 winners in the National Science Talent Search no one was more pleased—or less surprised—than Jim McKeehan, Dick's mathematics teacher. And McKeehan was justly proud: concurrent with the announcement came a letter of congratulation from the Science Clubs of America stating that Dick had listed his teacher as "the one person most influential in the development of my career."

Dick's project was in mathematical theory, and no one understands his talent and training in this field better than McKeehan. It is perhaps an understatement to say that the teacher has given it form and meaning. Last fall he asked Dick and two other boys to register for a special class of his, something new to the school and highly selective, to be conducted during McKeehan's assigned free period. This class, Dick says, provided the incentive. It changed him from just another "A" student into something of a youthful philosopher, a young thinker in the science of mathematics.

[1] Used by permission of the *Washington Education Journal* and reprinted from the April 1954 issue.

No traditionalism here

These daily tutorial sessions have never been concerned with the accepted scope of high school subject matter. McKeehan calls the course "What Is Mathematics: An Elementary Approach to Ideas and Methods," which is the title of the book the boys use (Courant and Robbins, Oxford University Press, 1941). It outlines the challenge to the class, and last fall the group worked successfully with material usually considered the province of college graduate study.

For example, the group has lounged around McKeehan's desk pondering and working with such topics as number theory, number systems, the algebraic theory of sets, geometrical constructions, classical problems in mathematics, projective, axiomatic, and non-Euclidean geometries and analytical geometry. The current semester includes functions and limits, limits and continuity, calculus of variations, and the calculus.

"They love it," says McKeehan. "They have never heard of it before and don't know it's supposed to be difficult. The boys have been seen studying their theories during other classes, and their

parents say the book is the only one they have ever brought home with real enthusiasm."

The schools are obligated

McKeehan's purpose is to stimulate and broaden the minds of superior students. He insists on the necessity of such a program and thinks the schools are obligated to expand it into other fields. The maturing young intellectual is too often and too sadly neglected. The type of student McKeehan brings to his special class very likely has been bored with the repetition of simplicities since the third grade. The public schools must adapt themselves to the requirements of this student's development or run the risk of characterizing his public education by a promiscuous waste of time and talent valuable to society. Intellectual inactivity can too easily convert brilliance into dull mediocrity, McKeehan feels. Dick is still not challenged to the extent he should be, his teacher believes; he should have been doing college work two years ago. Despite his national recognition today he has nevertheless neglected his talent by loafing along in the ordinary pursuits of a high school curriculum.

"Society can't afford this," says McKeehan.

To support his argument McKeehan presents another book, A. T. Bell's *The History of Mathematics*. The book suggests this question: how many great creative thinkers in mathematics can the United States boast of? The number is embarrassing, and most of those we can name received European educations. Most of them were already creative at Dick's age. They weren't entertaining themselves with high school subject matter. If we want intellectual leadership, we must do something effective about it in the public schools.

A classical approach

McKeehan has taught for 17 years, and for only two of these has he been able to conduct his special class. Administrative resistance to his ideas kept it out of the curriculum until he met Principal Dan Shovlin, who approved an experiment. Now the administration and faculty at Lake Washington are proud to acknowledge the value of McKeehan's approach and the wisdom of allowing him a free hand. They just wonder how he does it within the restrictions of a normal day—he teaches five mathematics classes and coaches a wrestling team. He thoroughly enjoys it all, however, and appreciates the opportunity he has. He hopes for a day when teachers will be given enough time to expand the program he has started.

Such teachers, he thinks, should be well versed in the theory of their fields. McKeehan's class is not an extension of high school math; it is not for college credit. But it gets under the conventional course material and into basic theory. Just such a classical approach is the crux of the school program he would like to see for his students, and it would require a respectable amount of advanced study and intellectual discipline of the teacher.

McKeehan is following the publications of the Ford Foundation projects with deep interest and hopes Washington can take a more active part in them. He is happy to hear of classes similar to his in other schools. He notes that in 1952, in Anacortes, an exceptional student in science named Fred MacLean was given a special program outside the curriculum in his senior year, and that Fred received an honorable mention in the Science Club's contest the year before Dick Fiddler entered. Other schools must be thinking in the same terms, and McKeehan, working with his boys each day, is glad that at least one question before American educators has a clear and definite answer.

NCTM

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Minutes of the Annual Business Session

Statler Hotel, Boston, Massachusetts

April 15, 1955

Mrs. Marie Wilcox called the meeting to order at 4:30 P.M.

I. Mr. Z. L. Loflin moved, seconded by Miss Catherine Lyons, that the minutes of the last meeting be approved as circulated. The motion carried.

II. Mr. M. H. Ahrendt, Executive Secretary, gave a preliminary financial report. He stated that the final report would not appear until after the close of the year which ends on June 1st. Note: By action of the Board annual financial reports will be published in the future.

III. Mr. Houston T. Karnes, Recording Secretary, gave a brief review of the actions of the Board during the past year. This review included items which were thought to be of interest to the members. The review follows:

(a) In view of the popularity of *The Arithmetic Teacher*, the Board at the December meeting made the following policy:
  (1) The annual dues for membership be $3.00 (three dollars) with the individual having a choice of the two journals.
  (2) For the sum of $5.00 (five dollars) the member would be entitled to both journals.

(b) *Nominating Procedure.* Also at the December meeting the Board set up the following policy on nominating procedure and elections:
  (1) The Committee on Nominations and Elections should be selected by the time of the Christmas meeting, one year in advance, and should have a preliminary meeting then, if possible. The Committee shall consist of nine members, with five members constituting a quorum.
  (2) Suggestions for nominations from the membership shall be secured in the following additional ways:
    (*a*) Enclose with the election ballot a request for suggestions for the following year.
    (*b*) Ask the Affiliated Groups to present suggestions through their delegates.
    (*c*) Request suggestions at the Delegate Assembly during the Annual Meeting.
    (*d*) Have a box at the Annual Meeting for the depositing of suggestions.
  (3) The Nominating Committee shall have at least two meetings for the screening of names and the selection of candidates.
    (*a*) The first meeting shall be held during the annual meeting.
    (*b*) The second meeting shall be held no later than September 10. Transportation expenses for the members of the Nominating Committee shall be paid for this meeting in accordance with the policy of paying expenses of Board members to the annual meeting.
  (4) At the second meeting the Nominating Committee shall decide on the system for a mail vote, if such a vote be necessary.
  (5) Nominations for directors shall be made so that there shall not be more than one director elected from each state; also, there shall be one director, and not more than two, elected from each area of the Affiliated Groups. (Secretary's note: This means that no state can have more than one director serving at any one time, and that all areas of the Affiliated Groups must have at least one director serving at all times. At most, there can be no more than two directors from every area of the

Affiliated Groups serving at any one time.)

(6) A copy of these procedures shall be given all members of the Nominating Committee every year.

(c) *Affiliated Groups.* The six affiliated group areas are now constituted as follows:

FAR WEST

Arizona	Oregon
California	Utah
Idaho	Washington
Montana	Wyoming
Nevada	

SOUTH WEST

Arkansas	New Mexico
Kansas	Oklahoma
Louisiana	Texas
Mississippi	Colorado

SOUTH EAST

Alabama	Virginia
Florida	West Virginia
Georgia	Maryland
South Carolina	District of Columbia
North Carolina	Delaware
Tennessee	

NORTH CENTRAL

Iowa	North Dakota
Michigan	South Dakota
Minnesota	Wisconsin
Nebraska	

CENTRAL

Illinois	Missouri
Indiana	Ohio
Kentucky	

NORTH EAST

New Jersey	Massachusetts
Pennsylvania	Vermont
New York	New Hampshire
Connecticut	Maine
Rhode Island	Ontario, Canada

(d) *Curriculum Committee.* The Board has set up two committees which will work together when deemed necessary. One committee is to study problems in grades one through six, and the other in grades seven through twelve. The personnel of the committees follows:

Grades 1–6	Grades 7–12
One-Year Term	*One-Year Term*
ANN PETERS	SHELDON MYERS
HERBERT SPITZER	BRUCE MESERVE
Two-Year Term	*Two-Year Term*
ALICE ROSE CARR	HOWARD FEHR
ESTHER J. SWENSON	JOHN MAYOR
Three-Year Term	*Three-Year Term*
FRED J. WEAVER	FRANK ALLEN
IRENE SAUBLE	CLIFFORD BELL

(e) *Special Publications Committee.* A special committee on publications has been authorized to study general problems and coordinate all Council publications. The committee is composed of the following: the three editors, chairman of the *Yearbook* planning committee, chairman of the supplementary publications committee, and one member from the Board to be appointed by the president. The Board member is to be chairman of the committee. Mr. Clifford Bell has been appointed by Mrs. Wilcox.

(f) *Financial Reports.* Annual financial reports will be published in THE MATHEMATICS TEACHER and *The Arithmetic Teacher.*

(g) *FSAF.* The Council is cooperating with FSAF in its work. Dr. H. F. Jamison will represent the Council in the conference to be held at San Jose State College on August 12–26, 1955.

(h) *Membership Drive.* The Board is sponsoring an all out membership drive. Its success will depend upon the effort of every member. The goal during the next year is a fifty per cent increase over the present state totals. (Which state or states will be the first to cross the goal line?)

(i) *Meetings.* The 1957 and 1958 annual meetings will be held in Philadelphia and Cleveland respectively. The 1956 Christmas meeting will take place at Jonesboro State College, Arkansas, and the 1957 summer meeting will be in Canada.

(j) *Yearbook.* Five new *Yearbooks* are in various stages of development. Two are well under way, one is just being planned and two have just been authorized.

(k) *NEA Building.* The members of the Council are greatly interested in NEA's new building since our central office is housed there. The Board authorized that a gift of $1,000 be given to NEA's building fund. (Note: Our central office has already moved into its new quarters.)

IV. Miss Agnes Herbert, Chairman of the Nominating Committee, reported on the election returns. Following her report Mrs. Wilcox introduced the new officers and directors. They are as follows:

Vice-President for College—FRANCIS G. LANKFORD, JR.

Vice-President for Junior High School—MILTON W. BECKMAN

Directors: IDA MAY BERNHARD—JACKSON B. ADKINS, and HENRY SWAIN

V. Mr. Ben A. Sueltz, Editor of *The Arithmetic Teacher*, was recognized by Mrs. Wilcox.

VI. Mr. Maurice Hartung, Chairman of the 1955–56 Nominating Committee, explained the

new procedure adopted by the Board at the meeting of December 27, 1954. Mr. Hartung suggested that those desiring to make recommendations to the committee should do so soon.

VII. Mr. W. A. Gager presented the following resolution:

> In view of the importance and all-prevailing application of mathematics in our present culture, the development of new mathematics so different from the traditional classical approach and so powerful in its methods of solution of problems, and the need for increased numbers of professional persons who can read, apply, and carry on research in the field of modern mathematics, be it resolved that The National Council of Teachers of Mathematics reaffirm its strong support of and cooperation in all efforts of the mathematical and scientific societies of America toward supplying this need.
>
> Respectfully submitted,
> *Howard F. Fehr*
> *Phillip S. Jones*
> *William A. Gager, Chairman*

Mr. Gager then moved the adoption of the resolution, which was seconded by Mr. Jackson B. Adkins. Mr. Howard Fehr moved that the resolution be amended to include the names of the following societies: The American Mathematical Society, The Mathematics Association of America, The National Scientific Foundation, and The American Association for the Advancement of Science. Mr. Adkins seconded the motion. The motion was duly adopted. The original motion with its amendment was then voted upon. The motion carried by one vote. Mr. Fehr moved, seconded by Mr. Gager, to reconsider the resolution. The motion passed. Mr. F. G. Lankford moved, seconded by Mr. Maurice Hartung, that the resolution be reviewed by the Board and submitted again at a later date. The motion carried.

VIII. The meeting was duly adjourned at 5:20 P.M.

Respectfully submitted,
Houston T. Karnes, Recording Secretary

Your professional dates

The information below gives the name, date, and place of meeting with the name and address of the person to whom you may write for further information. For information about other meetings, see the previous issues of THE MATHEMATICS TEACHER. Announcements for this column should be sent at least ten weeks early to the Executive Secretary, The National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D. C.

NCTM convention dates

CHRISTMAS MEETING

December 27–29, 1955
Sheraton-Park Hotel, Washington, D. C.
Veryl Schult, local chairman, District of Columbia Teachers College, Washington 9, D. C.

ANNUAL MEETING

April 11–14, 1956
Schroeder Hotel, Milwaukee, Wisconsin
Margaret Joseph, 1504 N. Prospect Avenue, Milwaukee 2, Wisconsin

JOINT MEETING WITH NEA

July 2, 1956
Portland, Oregon
Lesta Hoel, Public Schools, Portland, Oregon

SUMMER MEETING

August 19–22, 1956
University of California, Los Angeles, California
Clifford Bell, University of California, Los Angeles 24, California

Other professional dates

Fall Meeting of the California Mathematics Council, Southern Section
November 5, 1955
Manual Arts High School, 4131 S. Vermont Avenue, Los Angeles, California
Marian C. Cliffe, 450 N. Grand Avenue, Los Angeles 12, California

• NOTES FROM THE WASHINGTON OFFICE

Edited by M. H. Ahrendt, Executive Secretary, NCTM, Washington, D. C.

Below are reports of registrations for two recent meetings of the Council. These reports are especially significant because each shows a sharp increase in attendance. The average number of registrations at Christmas and Annual meetings during the previous three years had been 314 and 761 respectively. Thus the per cents of increase over the previous three-year averages are respectively 71 per cent and 38 per cent. It is hoped that these large increases represent the beginning of a trend that will make the conventions of the Council of benefit to an increasing number of teachers of mathematics.

Also given below is a report of memberships as of May 1, 1955. It might be interesting also to compare this figure with the membership count as of three years ago. On May 1, 1952, the total membership was 8733. The per cent of increase in the membership during the past three years is 17 per cent. Although this figure seems impressive, it represents an average increase of only 6 per cent per year. If all members of the Council will cooperate with the present membership drive, it should be possible for us to increase sharply the rate of growth of the National Council.

Registrations at the Fifteenth Christmas Meeting

The National Council of Teachers of Mathematics, St. Louis, Missouri, December 27–29, 1954

Alabama	2	Nebraska	5
Arizona	2	New Hampshire	1
Arkansas	11	New Jersey	9
California	3	New Mexico	1
Connecticut	3	New York	7
District of Columbia	5	North Carolina	1
Florida	3	Ohio	23
Georgia	2	Oklahoma	12
Idaho	1	Pennsylvania	5
Illinois	135	Rhode Island	1
Indiana	18	South Carolina	2
Iowa	13	Tennessee	10
Kansas	11	Texas	6
Kentucky	1	Utah	2
Louisiana	7	Virginia	1
Maryland	1	West Virginia	3
Massachusetts	4	Wisconsin	11
Michigan	10	Foreign	2
Minnesota	7		
Mississippi	1	Total	537
Missouri	195		

Registrations at the Thirty-third Annual Meeting

The National Council of Teachers of Mathematics, Boston, Massachusetts, April 13–16, 1955

Alabama	1	District of Columbia	10
California	4	Florida	9
Colorado	2	Georgia	5
Connecticut	62	Illinois	25
Delaware	1	Indiana	11

Iowa	6	Pennsylvania	28
Louisiana	5	Rhode Island	22
Maine	27	South Carolina	1
Maryland	11	Tennessee	3
Massachusetts	526	Texas	6
Michigan	19	Vermont	8
Minnesota	3	Virginia	13
New Hampshire	63	Washington	2
New Jersey	36	West Virginia	1
New York	89	Wisconsin	11
North Carolina	1	Canada	11
Ohio	23		
Oklahoma	1	Total	1,047
Oregon	1		

Membership in The National Council of Teachers of Mathematics, May 1, 1955

	Individual	*Institutional*	*Total*		*Individual*	*Institutional*	*Total*
Alabama	71	37	108	New Jersey	288	78	366
Arizona	26	14	40	New Mexico	42	19	61
Arkansas	71	21	92	New York	662	201	863
California	364	215	579	North Carolina	126	46	172
Colorado	120	23	143	North Dakota	20	11	31
Connecticut	140	40	180	Ohio	400	80	480
Delaware	39	6	45	Oklahoma	117	46	163
District of Columbia	115	16	131	Oregon	74	35	109
Florida	192	45	237	Pennsylvania	474	181	655
Georgia	111	34	145	Rhode Island	41	9	50
Idaho	7	5	12	South Carolina	45	38	83
Illinois	662	102	764	South Dakota	25	8	33
Indiana	309	43	352	Tennessee	113	41	154
Iowa	151	51	202	Texas	338	142	480
Kansas	182	35	217	Utah	28	15	43
Kentucky	53	20	73	Vermont	20	5	25
Louisiana	137	39	176	Virginia	201	49	250
Maine	44	8	52	Washington	129	38	167
Maryland	175	32	207	West Virginia	66	11	77
Massachusetts	267	56	323	Wisconsin	246	61	307
Michigan	297	103	400	Wyoming	19	8	27
Minnesota	174	72	246				
Mississippi	60	23	83	TOTALS	7,596	2,261	9,857
Missouri	187	51	238	U. S. Possessions	29	14	43
Montana	41	9	50	Canada	80	62	142
Nebraska	82	23	105	Foreign	118	96	214
Nevada	11	6	17				
New Hampshire	34	10	44	GRAND TOTALS	7,823	2,433	10,256

The abacus and the brain

Continued from page 417

ernments are imposed upon him than on what values he creates.

Distrust of our mechanical age, fear that men will be destroyed by the engines which he has devised, is so widespread today that it has developed its own cant. But it often happens that men's fate overtakes them in the one way they had not sufficiently feared, and it may be that if we are to be destroyed by the machine it will not be in quite the manner we have been fearfully envisaging. Perhaps we are in no greater danger of being blown up by the atom bomb than we are of being destroyed by a wrong understanding of the abacus.

• AFFILIATED GROUPS

by H. Glenn Ayre, Chairman, Western Illinois State College, Macomb, Illinois

The purpose of this letter is to state some of the accomplishments of the past year and to mention some of the activities being planned for 1955–56.

The growth and importance of the Affiliated Groups are reflected in recent Board action which gives the affiliates greater responsibility and more active participation in the administration of the Council.

Elections

At the St. Louis meeting, December, 1954, the Board approved a recommendation by the Committee on Nominations which ask the Affiliated Groups to assist in the selection of candidates for officers and board members of the Council by submitting in writing through their delegates the names of individuals they would like the Nominating Committee to consider for the ballot. This is certainly an important step in the implementation of the democratic process, and should result in a fine service to the Committee on Nominations and to the selection of our most capable people for offices, and at the same time should give the Affiliated Groups a stronger voice in the business affairs of the Council.

In order to provide for geographic distribution of board members, it was voted that there shall be not more than one member of the Board elected from any one state; also, there shall be one director, and not more than two, elected from each of the six affiliated regions. This means that no state shall have more than one director serving at any one time, and that all regions of the Affiliated Groups must have at least one director serving at all times, but not more than two. None of these geographic regulations applies to officers of the Council—only to members of the Board of Directors.

Six affiliated regions

Action initiated at St. Louis and approved at Boston, April, 1955, establishes six affiliated regions, and provides for a rotation of officers so that the chairman shall serve for a term of three years and the regional representatives shall likewise serve for three years with two representatives to be appointed each year. The six regions, the states in the regions, the representatives, and the number of Affiliated Groups in each region at the present time are:

Northeastern Region—11 Affiliated Groups

Representative: Catherine A. V. Lyons, 12 South Fremont Avenue, Pittsburgh 2, Pennsylvania

States and Canadian Province: Connecticut, Maine, Massachusetts, New Hampshire, New Jersey, New York, Ontario, Pennsylvania, Rhode Island, Vermont

Southeastern Region—17 Affiliated Groups

Representative: Houston Banks, George Peabody College for Teachers, Nashville, Tennessee

States and District: Alabama, Delaware, District of Columbia, Florida, Georgia, Maryland, North Carolina, South Carolina, Tennessee, Virginia, West Virginia

Central Region—12 Affiliated Groups

Representative: Adeline A. Riefling, 3507 Hawthorne Boulevard, St. Louis 4, Missouri

States: Illinois, Indiana, Kentucky, Missouri, Ohio

North Central Region—7 Affiliated Groups

Representative: Irene Sauble, Detroit

Geographic Division of the Six Affiliated Regions

Public Schools, 467 West Hancock, Detroit 1, Michigan

States: Iowa, Michigan, Minnesota, Nebraska, North Dakota, South Dakota, Wisconsin

Southwestern Region—12 Affiliated Groups

Representative: Ida May Bernhard, Texas Education Agency, Austin, Texas

States: Arkansas, Colorado, Kansas, Louisiana, Mississippi, New Mexico, Oklahoma, Texas

Western Region—6 Affiliated Groups

Representative: Elizabeth Roudebush, Seattle Public Schools, 815 Fourth Avenue, North, Seattle 9, Washington

States: Arizona, California, Idaho, Montana, Nevada, Oregon, Utah, Washington, Wyoming.

The division of territory was more or less arbitrary; however, consideration was given to such factors as area, density of population, membership in NCTM, and number of Affiliated Groups. During the past year the Greater Toledo Council of Teachers of Mathematics affiliated with The National Council, making a total of 65 groups, representing approximately 11,000 members affiliated with The National Council. The growth in stature and scope of the Council is an indication that the Affiliated Groups have made an important contribution to the expanding services of the Council. It is hoped that each organization will work closely with the regional representative in order to increase its services to the local community and to The National Council.

Handbook

The work which was started on the *Handbook for Organizations of Mathematics Teachers* has been completed, and the *Handbook* was presented to the Sixth Delegate Assembly in Boston. The job of edition was taken over by Henry Swain last summer and carried to a successful completion. The Assembly voted to give

one copy to each Affiliated Group and to make additional copies available through the Washington Office at fifty cents per copy. It is hoped that all groups will find this publication helpful in their programs. We solicit your suggestions and criticisms for possible revision.

Annual dues of affiliation

By vote of the First Delegate Assembly in 1950 it was agreed that each affiliated group should be required to renew affiliation and pay dues between October 1 and December 31 each year. Forms for this annual renewal are now available, and will be sent to each group in September. It would please your chairman very much if each group would find it possible to comply with this policy. At the present time, renewal and payment of dues are scattered over the entire school year. There is also a provision for the waiver of dues for any organization having 75 per cent of its members also members of The National Council.

At the Second Delegate Assembly the delegates voted to make payment of dues a requirement for eligibility to vote in the delegate assembly.

Annual reports

New forms have been prepared for the annual reports. These forms will be mailed to you by your regional representative. They should be filled out promptly, one copy returned to your representative, and one copy to the chairman of Affiliated Groups.

Your delegate and alternate to the seventh delegate assembly

It is suggested that at your first meeting this fall each group select an official delegate and alternate to represent you at the Seventh Delegate Assembly to be held at the annual meeting in Milwaukee next April. Send these names to your regional representative and the chairman of the Committee on Affiliated Groups promptly.

Newsletter

The assembly at Boston voted to resume the publication of the *Affiliated Newsletter*. Mrs. May Kelly, 32 North 29th Avenue, Longport, New Jersey, has been appointed editor. She needs your support. Will you send to her news items about your members, programs, and special projects? Make a special effort to furnish her with copies of your local Newsletters and other publications.

It is now planned to publish four issues during 1955–56. A limited number of copies (probably four or five) will be sent to each Affiliated Group. However, additional copies will be available at fifty cents for the year, provided Mrs. Kelly receives your subscription early enough to place the printing order.

Membership

The membership drive under the chairmanship of Miss Mary Rogers is now in full swing. We are appealing to the Affiliated Groups for their strong and active support in this important project. The goal has been set at 15,000 members of NCTM by spring 1956. This is an ambitious undertaking, to put it modestly. If each of the 65 Affiliated Groups would organize a membership committee of active enthusiastic workers, this goal could be realized without question. That is exactly what your Committee on Affiliated Groups would urge you to do. It takes personal contacts to enlist teachers for the cause of better mathematics in our schools. Can we count on you? We now have an added attraction to join NCTM in the fact that teachers can now have a choice of The Mathematics Teacher or *The Arithmetic Teacher* with a membership fee of $3.00. Those desiring both journals may have them with a membership fee of $5.00.

Don't Miss These Successful New Publications

RECREATIONAL MATHEMATICS, by W. L. Schaaf

This bibliography, probably the most comprehensive ever published, lists recreational materials in 58 categories under the following main headings: general works, arithmetical and algebraic recreations, geometric recreations, assorted recreations, magic squares, the Pythagorean relationship, famous problems of antiquity, and mathematical miscellanies. 136 pages. $1.20 each.

GEOMETRY GROWING, by W. R. Ransom

An interesting collection of materials. Presents "bits of geometry growing" from Pythagoras to Newton. Valuable for enrichment. Throws new light on many famous theorems. 40 pages. 75¢ each.

BYROADS OF ALGEBRA, by Margaret Joseph

Gives enrichment material on the level of high-school algebra. Discusses algebra to simplify arithmetic computations, number puzzles, and algebraic acrobatics. Useful for class or club. 16 pages. 40¢ each.

HOW TO STUDY MATHEMATICS, by Henry Swain

A popular seller, this handbook was prepared for high-school students and written in their language. Contains many practical suggestions for succeeding in homework, classwork, taking tests, and the like. Gives special suggestions for studying some of the difficult areas in high-school mathematics. Illustrated. 32 pages. 50¢ each.

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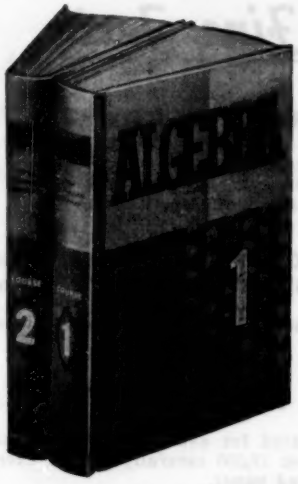

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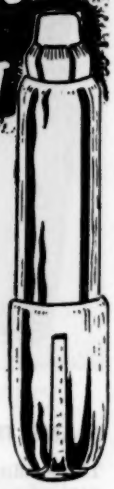

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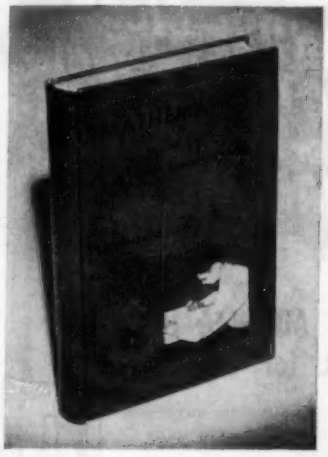

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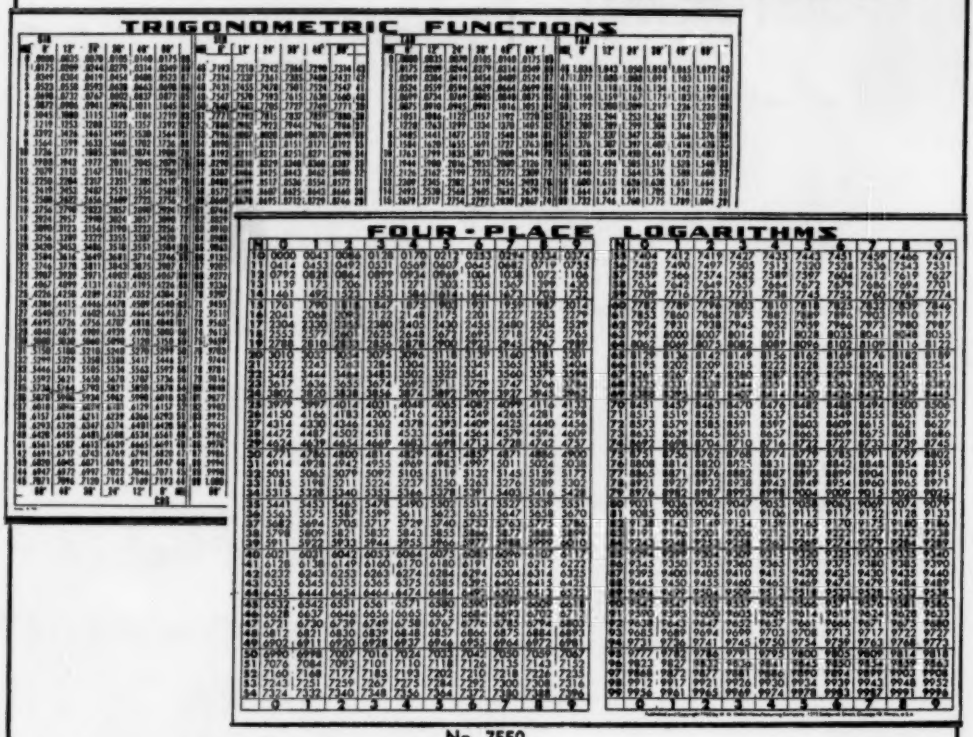

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